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Energy release rate and phase angle of delamination in sandwich beams and symmetric adhesively bonded joints

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A B S T R A C T

Delamination in sandwich structures along the interface between the face sheet and the core, or along the adherend/adhesive interface in adhesively bonded joints, is one of the most common failure modes of this type of tri-layer structure. This delamination is usually modeled as an interface crack problem, for which the energy release rate and phase angle can be calculated using interface fracture mechanics solutions. Existing interface fracture mechanics solutions, however, ignore the effect of transverse shear deformation, which can be significant for short crack. In an effort to overcome this shortcoming, this study presents new analytical solutions for the energy release rate and for the phase angle of the interface crack in sandwich structures or adhesively bonded joints. Since the new solutions incorporate relative rotation at the tip of the delamination, transverse shear effects are taken into account in this study. Typical delaminated sandwich and adhesively bonded joint specimens are analyzed by using the new solutions, as well as by the existing solutions. The energy release rate predicted by the present model agrees very well with that predicted by FEA, and furthermore it is considerably more accurate relative to existing models. As the existing model neglects the transverse shear force, it underestimates the total energy release rate. A stress field analysis is also conducted in this study in order to clarify some misunderstandings in the literature on the determination of the phase angle of adhesively bonded joints using an interface stress-based method.

1. Introduction

Sandwich structures and adhesively bonded joints are two types of tri-layer structures widely used in many industries, including aerospace, automotive (Noor et al., 1996), and civil infrastructure (Davalos et al., 2001; Qiao and Wang, 2005a,b). A sandwich structure consists of a thick, low-density core material, and two thin, stiff, and strong face sheets. This configuration improves flexural strength and provides high stiffness-to-weight ratio. An adhesively bonded joint consists of two adherends and a very thin layer of adhesive. Compared to other structural joints, adhesively bonded joints have higher structural efficiency, lower stress concentration, and better fatigue endurance. Interface debonding (the face sheet/core interface debonding in sandwich structures and the adherend/adhesive interface debonding in adhesively bonded joints) is one of the most common failure modes of this type of structure.

The face sheet/core delamination of sandwich structures has been studied extensively, typically by interface fracture mechanics (Zenkert, 1989; Cantwell and Davies, 1995; Prasad and Carlsson, 1994; Cantwell et al., 1999; Ural et al., 2003; Østergaard and Sørensen, 2007). In order to retrieve interface fracture parameters (the energy release rate (ERR) and its phase angle) from experimental data and evaluate the potential for delamination in sandwich constructions, both numerical methods such as finite element analysis (FEA) (Prasad and Carlsson, 1994) and analytical solutions (Østergaard and Sørensen, 2007) were commonly used. The former approach is not convenient or efficient due to stress oscillation at the interface crack tip. The later method is much more desirable due to its simplicity and ease of application. The existing analytical solution (Østergaard and Sørensen, 2007) is based on the classical interface fracture solution of bi-layers (Suo and Hutchinson, 1990); however, the major drawback of this solution is that the effect of transverse shear is not considered (Østergaard and Sørensen, 2007).

To analyze the debonding of adhesively bonded joints, two major approaches have been adopted: a strength of materials approach and a fracture mechanics approach. The former approach, which has been used for over seven decades, focuses on the prediction of interfacial peel and shear stresses within the adhesive layer (Goland and Reissner, 1944; Delale et al., 1981; Wang and Zhang, 2009). In the latter approach, which has been used more recently (Krenk, 1992; Alfredsson and Hogberg, 2007; Fernlund, 2007; Shahin and Taheri, 2008), two fracture mechanics parameters, ERR and its phase angle, are calculated. Three methods are

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commonly used to calculate the ERR and its phase angle: (1) a finite element analysis (FEA); (2) a classical interface fracture solution (Suo and Hutchinson, 1990; Østergaard and Sørensen, 2007); and (3) an interface stress-based method (Krenk, 1992; Alfredsson and Hogberg, 2007; Fernlund, 2007; Shahin and Taheri, 2008). In the interface stress-based method, the maximum interface peel and shear stresses at the delamination tip (within the adhesive layer) are first obtained using the aforementioned strength of materials method. The ERR in mode I or II is then calculated as half the product of the square of the peel stress (or shear stress) at the crack tip and the stiffness of peel (or shear). This approach was originally developed for symmetric adhesive bonded joints (Krenk, 1992), and recently it has been used for asymmetric joints by a few researchers (Alfredsson and Hogberg, 2007; Fernlund, 2007; Shahin and Taheri, 2008). However, the validation of such an extension is questionable because the phase angle obtained through the interface stress-based method is not reliable for an asymmetric joint (Alfredsson and Hogberg, 2007).

To characterize interface fracture properties of sandwich structures and adhesively bonded joints, a number of experimental methods have been developed. Beam-type specimens are commonly used in these experiments. For sandwich construction, these specimens include the Crack Sandwich Beam (CSB) specimen for Modes I and II fracture testing (Carlsson and Prasad, 1993), the Double Cantilever Beam (DCB) specimen (Prasad and Carlsson, 1994), the Three-Point Bending Specimen (TPBS) (Cantwell et al., 1999), and the Tilted Sandwich Debond (TSD) specimen (Li and Carlsson, 1999). For adhesively bonded joints, typical experimental specimens include the DCB specimen, the End-Notched Flexure (ENF) specimen, and the Mixed-Mode Bending (MMB) specimen.

This study presents a new interface fracture mechanics analysis for a general symmetric tri-layer beam. New closed-form solutions for the ERR and its phase angle are obtained; these solutions are applicable to interface delamination in sandwich constructions or adhesively bonded joints. As pointed out by Østergaard and Sørensen (2007), Transverse force is of major importance in many practical applications of sandwich beams. The new solutions represent a significant improvement on the old methods, stemming from the inclusion of transverse shear effects. Recent studies (Qiao and Wang, 2005c; Wang and Qiao, 2004a,b,c) show that the key quantity in the inclusion of transverse shear effect (in the interface fracture analysis) is the crack tip rotation. The existing classical solution (Suo and Hutchinson, 1990) assumes that the cross-section of a bi-layer structure at the crack tip remains on one plane after deformation under external loads. This assumption leads to zero crack tip rotation. As a result, the classical solution may not include the transverse shear effect.

Many methods have been proposed to estimate crack tip rotation, including finite element analysis calibration (Li et al., 2003), the sub-layers method (Zou et al., 2001), and beam on elastic foundation model for symmetric specimens (Kanninen, 1973). General closed-form solutions for the deformation at the tip of an interface crack between two shear deformable layers have been developed recently (Wang and Qiao, 2004a; Qiao and Wang, 2004), based on novel bi-layer beam models. In these solutions, each layer of the structure rotates independently, thus capturing the relative deformation of the crack tip (Qiao and Wang, 2005c). In order to capture the effects of the transverse shear forces in sandwich structures and adhesively bonded joints, this study extends the two bi-layer beam models mentioned earlier (Wang and Qiao, 2004a; Qiao and Wang, 2004) to tri-layer construction.

It should be pointed out that all of the aforementioned studies, and the present work, are limited to linear elastic behavior. For typical sandwich structures or adhesively bonded joints with tough structural adhesives, a long plastic deformation zone may develop ahead of the crack tip prior to crack propagation (Chiang and Chai, 1993). In this case, nonlinear fracture mechanics based on a cohesive zone model (Chai, 2003) may be more appropriate. Therefore, new fracture experimental methods that incorporate the cohesive zone behavior of the structure may be used (Swadener and Liechti, 1998; Chai, 2003).

2. Interfacial fracture mechanics of delamination in symmetric tri-layer beams

Both sandwich beams and symmetric adhesively bonded joints can be modeled as symmetric tri-layer beams. For the symmetric tri-layer beam, shown in Fig. 1(a), the top and bottom layers (face sheets or adherends) are made of same material and same geometry. The mid-layer is either much thicker (sandwich beam) or thinner (adhesively bonded joint) than the top and bottom layers. Consider a delamination occurring along the interface between the top layer and the mid-layer. As shown in Fig. 1(a), the top layer is modeled as Beam 1, and the mid-layer and bottom layer together are modeled as a composite Beam 2. The length of the uncracked region L in Fig. 1(a) is relatively large compared to the thickness of the whole beam $H = h_1 + h_2 + h_3$, so the boundary effect of the intact end of the structure is negligible. This configuration represents a small crack tip element of a delaminated tri-layer mean, where the cracked and uncracked portions are joined, and to which the generic loads, previously determined by a global beam analysis, are applied.

By using Timoshenko's beam theory, we can express the deformations of Beams 1 and 2 as:

$$U_i(x, z_i) = u_i(x) + z_i\phi_i(x), \tag{1}$$

$$W_i(x, z_i) = w_i(x), \tag{2}$$

where subscript $i = 1, 2$, represent Beams 1 and 2 in Fig. 1(a), respectively. $u_i(x)$ and $w_i(x)$ are the longitudinal and transverse displacements of the neutral axes of Beam i, respectively. $\phi_i(x)$ represents the rotations of Beam i.

The constitutive equations of Beam i are given by:

$$N_i(x) = C_i\frac{du_i(x)}{dx}, \quad Q_i(x) = B_i\left(\phi_i(x) + \frac{dw_i(x)}{dx}\right),$$
$$M_i(x) = D_i\frac{d\phi_i(x)}{dx}, \tag{3}$$

where $N_i(x)$, $Q_i(x)$, and $M_i(x)$ are the resultant axial force, transverse shear force, and bending moment of Beam i, respectively. C_i, B_i, and D_i are the axial, shear, and bending stiffnesses of Beam i, respectively. They are given below for plane stress conditions:

$$C_1 = bE_1h_1, \quad B_1 = \kappa bG_1h_1, \quad D_1 = \frac{E_1bh_1^3}{12},$$
$$C_2 = E_2bh_2 + E_3bh_3, \quad B_2 = \kappa G_2bh_2 + \kappa G_3bh_3,$$
$$D_2 = \frac{E_2b}{3}[(h_2-d)^3 + d^3] + \frac{E_3b}{3}[(h_3+d)^3 - d^3]. \tag{4}$$

Here E_i, G_i, are the longitudinal and shear moduli, respectively, of the top layer ($i = 1$), mid-layer ($i = 2$), and the bottom layer ($i = 3$). κ is the shear correction coefficient, which was set to 5/6 for this study. $d = \frac{E_2h_2^2 - E_3h_3^2}{2(E_2h_2 - E_3h_3)}$ is the distance of the neutral axis of Beam 2 to the bottom of the midlayer. b is the width of the beam. N_{10}, N_{20}, Q_{10}, Q_{20}, M_{10}, and M_{20} are the applied axial force, transverse shear force, and bending moment, respectively, at the crack tip. N_T, Q_T, and M_T are the total applied resultant axial force, shear force, and bending moment about the neutral axis of composite Beam 2, given by

$$N_T = N_{10} + N_{20}, \quad Q_T = Q_{10} + Q_{20},$$
$$M_T = M_{10} + M_{20} + N_{10}\left(\frac{h_1}{2} + h_2 - d\right). \tag{5}$$