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Structural vibration of flexural beams with thick unconstrained layer damping

Fernando Cortés *, María Jesús Elejabarrieta

Department of Mechanical Engineering, Mondragon Unibertsitatea, Loramendi 4, 20500 Mondragon, Gipuzkoa, Spain

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ABSTRACT

In this paper, the dynamic behaviour of free layer damping beams with thick viscoelastic layer is analysed. A homogenised model for the flexural stiffness is formulated employing Reddy and Bickford's quadratic shear in each layer, in contrast to the classical model of Oberst and Frankenfeld for thin beams, which does not take into account shear deformations. The results provided by these two models in free and forced vibration are compared by means of finite element procedures with those of a 2D model, which considers extensional and shear stress, and longitudinal, transverse and rotational inertias.

The viscoelastic material is characterised by a fractional derivative model, which takes into consideration the variation of the complex modulus with frequency. To avoid the frequency dependence of the stiffness matrices, the extraction of the eigenvalues and eigenvectors is completed by a new iterative method developed by the authors. The frequency response to a harmonic force is deduced by the superposition of modal contribution functions.

From these numerical applications it can be concluded that the model for thick beams provides sufficient accuracy for practical applications, able to reproduce the mechanical behaviour of free layer damping beams with thick viscoelastic layer, reducing the storage needs and computational time with respect to a 2D model.

1. Introduction

The unconstrained layer damping configuration, also known as free layer damping (FLD), is a technology commonly employed in passive control techniques using viscoelastic materials (see e.g. Nashif et al. (1985), Sun and Lu (1995) or Jones (2001) for details). In this configuration, the damping material is bonded on vibrating flexural structural elements, normally made of metallic materials, as Fig. 1 illustrates.

In Fig. 1 the cross-section of a two-layer composed beam is represented, where the thickness of the base and the viscoelastic layers are H and H_1, respectively, and the width of the beam is represented by b. The properties of the elastic material are Young's modulus E, Poisson's coefficient ν and density ρ, and those of the viscoelastic are the complex extensional modulus E_1^*, ν_1 and ρ_1, respectively. If both materials are considered isotropic, the shear modulus G and G_1^* are related with the extensional by $G = E/2(1 + \nu)$ and $G_1^* = E_1^*/2(1+\nu_1)$, for the base and damping materials, respectively.

The complex nature of the modulus of viscoelastic materials is due to their ability to dissipate mechanical energy. In effect, a complex relationship between stress and strain allows to represent the hysteretic behaviour of this kind of materials (Myclestad, 1952). This dissipative property may be described by the loss factor η_1

* Corresponding author. Tel.: +34 943794700; fax: +34943791536.
E-mail addresses: fcortes@eps.mondragon.edu (F. Cortés), mjelejabarrieta@eps.mondragon.edu (M.J. Elejabarrieta).

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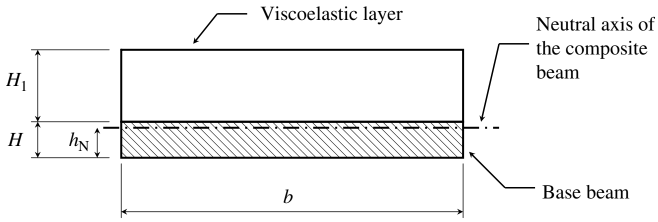

Fig. 1. Cross-section of a two-layer beam.

$$\eta_1 = \frac{E_1''}{E_1'}, \tag{1}$$

where E_1' and E_1'' are the real and imaginary components of the complex modulus, $E_1^* = E_1' + iE_1''$, which are known as the storage and the loss modulus, respectively, the former being also frequently represented by E_1. For polymers, both storage and loss modulus, and consequently loss factor, vary with temperature and frequency depending on three different behaviours: rubbery, vitreous and of transition (see e.g. Ward and Hadley, 1993). In this sense, Jones (2001) reviews experimental data for complex modulus of some typical viscoelastic solids, including elastomers, adhesives and specially compounded materials. The experimental characterisation of low stiffness damping materials, which are not appropriate to prepare self-supporting test specimens, may be achieved by means of the ASTM E 756-04 (2004) standard, wherefrom the nomenclature has been taken.

The classical study of thin FLD beams is derived following Oberst and Frankenfeld (1952), who, based on Euler–Bernoulli's beam theory, provide an equivalent homogenised complex flexural stiffness B_{eq}^* given by

$$B_{eq}^* = B^* + B_1^*, \tag{2}$$

where $B^* = EI^*$ and $B_1^* = E_1^* I_1^*$ are the complex flexural stiffness of the base and damping layers, respectively, and I^*, I_1^* are the complex cross-sectional second order moments, given by

$$I^* = \frac{1}{12} bH^3 + bH(h_N^* - H/2)^2 \tag{3}$$

and

$$I_1^* = \frac{1}{12} bH_1^3 + bH_1(H + H_1/2 - h_N^*)^2, \tag{4}$$

respectively, in which the complex position of the neutral axis h_N^* is represented by

$$h_N^* = \frac{EH^2 + E_1^* H_1(2H + H_1)}{2(EH + E_1^* H_1)}. \tag{5}$$

The complex character of these geometric properties signifies that they are not constant during the vibration.

Oberst and Frankenfeld (1952) do not contemplate the use of thick layers, in which the shear effects may be considerable and cannot be neglected. For a beam made of a unique elastic material, different models may be used for shear consideration, such as the based on Timoshenko's or Reddy–Bickford's theories (see Wang et al. (2000) for more details). The former induces a constant shear strain along thickness that does not satisfy the boundary conditions on the surface, thus, a shear correction factor is commonly employed. The latter involves a quadratic distribution confirming the force equilibrium, and thus the shear correction factor is not needed.

In short, in this paper the structural vibration reduction using FLD treatments with thick viscoelastic layer is analysed by finite element procedures, taking into account shear effects. Therefore, next an equivalent complex flexural stiffness is derived considering quadratic shear stress based on Reddy–Bickford's theory. Later, numerical applications for cantilever beams are presented comparing the solutions provided by three different finite element models: a 2D plane model, a 1D beam model that employs the Oberst and Frankenfeld theory for thin beams and does not consider the shear, and another 1D beam model that uses the homogenised stiffness developed here for thick layers. The damping material is characterised by means of a fractional derivative model involving the frequency dependence of the complex modulus, which implies important disadvantages for the dynamic analysis. Thus, the extraction of the eigenvalues and eigenvectors is carried out by a simple and effective iterative algorithm developed by Cortés and Elejabarrieta (2006) employing the undamped eigensolutions and the eigenvector derivatives. Finally, the dynamic response of the three models is compared using the modal contribution functions (MCF) superposition method (Cortés and Elejabarrieta, 2005).

2. Homogenised model for a two-thick layer beam

Next the theoretical study of a two-layer beam is presented, in which quadratic shear stress is taken into account. An equivalent flexural stiffness will be deduced for pinned–pinned beams, considering that all materials are purely elastic.