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Constitutive relations in 3-D for a wide range of strain rates and temperatures – Application to mild steels

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Abstract

An original phenomenological thermo-visco-plastic model is reported that encompasses strain hardening, strain rate and temperature sensitivity. The model is based to some extent on the concept of physical modeling proposed earlier by Klepaczko [Klepaczko, J.R. 1975. Thermally activated flow and strain rate history effects for some polycrystalline FCC metals. Mater. Sci. Engng. 18, 121–135] , and also by different authors, for example: [Becker, R. 1925. Uber die plastizitat amorpher und kristalliner fester korper. Z. Physik 26, 919–925; Seeger, A., 1957. Dislocations and Mechanical Properties of Crystals, Wiley, New York; Conrad, H., 1964. Thermally activated deformation of metals J. Metals 16, 582; Gilman, J.J. 1968. Dislocation dynamics and response of materials to impact Appl. Mech. Rev. 21, 767–783; Gibbs, G.B., 1969. Thermodynamic analysis of dislocation glide controlled by dispersed local obstacles. Mater. Sci. Engng. 4, 313–328; Kocks, U.F., Argon, A.S., Ashby, M.F., 1975. Thermodynamics and kinetics of slip. In: Progress in Materials Science, vol. 19. Pergamon Press, New York, p. 19; Kocks, U.F. 1976. Laws for work-hardening and low-temperature creep. J. Eng. Mater. Technol. 98, 76–85, and later by many others.

The thermo-visco-plastic formulation, called RK and applied in this paper, has been verified experimentally for strain rates of $10^{-4}\ \text{s}^{-1} \leqslant \dot{\bar{\varepsilon}}_\text{p} \leqslant 5\times 10^3\ \text{s}^{-1}$ and temperatures $213\ \text{K} \leqslant T \leqslant 393\ \text{K}$, it covers the range of dynamic loadings observed during crash tests and other impact problems. In order to implement the RK constitutive relation, a thermo-visco-plastic algorithm based on the J_2 theory of plasticity is constructed. The type of algorithm is a return mapping one that introduces the consistency condition ($f = \bar{\sigma} - \sigma_y, f = 0$), without the overstress state proposed by Perzyna [Perzyna, P. 1966. Fundamental problems in viscoplasticity. Advances in Applied Mechanics, vol. 9. Academic Press, New York, pp. 243–377]. The coupling of the RK constitutive relation with the integration scheme of the thermo-visco-plastic algorithm has demonstrated its efficiency for numerical analyses of different dynamic processes such as Taylor test [Zaera, R., Fernández-Sáez, J. 2006. An implicit consistent algorithm for the integration of thermoviscoplastic constitutive equations in adiabatic conditions and finite deformations. Int. J. Solids Struct., 43, 1594–1612.], ring expansion [Rusinek, A., Zaera, R., 2007. Finite element simulation of steel ring fragmentation under radial expansion. Int. J. Impact Eng. 34, 799–822], dynamic tension test [Rusinek, A., Zaera, R., Klepaczko, J.R., Cherigueme, R., 2005. Analysis of inertia and scale effects on dynamic neck formation during tension of sheet steel. Acta Mater. 53, 5387–5400], perforation of metallic sheets [Rusinek, A., 2000, Modélisation thermoviscoplastique d'une nuance de tôle d'acier aux grandes vitesses de déformation. Etude expérimentale et numérique du cisaillement, de la traction et de la perforation, Ph.D. thesis, University of Metz,

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France], and other cases. All the equations are implemented via the user subroutine VUMAT in the ABAQUS/Explicit code for adiabatic conditions of plastic deformation.

Keywords: Mild steel; RK Constitutive relation; Thermo-visco-plasticity; Consistent algorithm; Dynamic plasticity; Numerical simulations; Finite element code

1. Introduction

Behavior of materials at high strain rates and at different temperatures is of high priority in the sphere of crashworthiness. In order to understand the evolution of deformation fields and the mechanisms of failure at high loading rates, precise analytical and numerical tools are required. In this study, the prime interest is the quasi-static and dynamic behavior of the mild steel ES used in the automotive industry. The study of a specific material for a well-determined application, in this case the automobile crash, circumscribes the work area in terms of the effective stress $\bar{\sigma}$, the effective plastic deformation $\bar{\varepsilon}_p$, the effective strain rate $\dot{\bar{\varepsilon}}_p$, and the temperature T (see Table 1). The reason for developing an advanced phenomenological approach is the ease with which this type of constitutive relation $\sigma = f(\bar{\varepsilon}_p, \dot{\bar{\varepsilon}}_p, T)$ may be introduced into finite element codes such as ABAQUS (2004). It is assumed that an adequate precision in material characterization can be achieved, as in the case of a physical formulation in which, for example, the evolution of dislocation density is taken into account. The difficulties of applying the physical formulation to finite element codes lies in a set of differential equations introduced to follow the evolution of the microstructure defined by one or more internal variables, then the constitutive relation is more complicated, for example $\sigma = f[\bar{\varepsilon}_p, \dot{\bar{\varepsilon}}_p, T, S_j(\bar{\varepsilon}_p, \dot{\bar{\varepsilon}}_p, T)]$, where S_j is a set of j internal state variables. It is clear however that with the phenomenology approach a simplification can be arranged more precisely on the basis of physical notions leading to a constitutive relation of the form $\bar{\sigma} = f(\bar{\varepsilon}_p, \dot{\bar{\varepsilon}}_p, T)$. This is an approach defined as the Mechanical Equation of State (MES) with absence of strain rate and temperature history effects, Klepaczko and Duffy (1982). Here, a comparison is also made between the proposed constitutive relation, called RK, and other formulations, such as Johnson and Cook (1983) or Cowper and Symonds (1952), implemented originally into commercial finite element codes. However, many others constitutive relations are frequently used to simulate dynamic processes. The most common approach is that derived via analogy to dislocation dynamics, for example Zerilli and Armstrong (1987). The previous list is not exhaustive and comparison between several formulations was discussed by Liang and Khan (1999).

Another group of constitutive relations, without assumption of the MES, is based on hypothetical evolution of the rate of strain hardening, $\theta = g(\varepsilon_p, \dot{\bar{\varepsilon}}_p, T)$, where $\theta = d\sigma/d\bar{\varepsilon}_p$. Some typical examples in that group are constitutive relations introduced for example by Bodner and Partom (1975), Kocks (1976), Klepaczko (1987, 1994), more recently by Molinari and Ravichandran (2005) and also others. The best example is the Mechanical Threshold Stress (MTS) model, Follansbee and Kocks (1988). The main problem with this concept is a serious difficulty in formulation of a precise explicit mathematical form of the evolution of strain hardening rate θ with plastic strain, strain rate and temperature. Even if this task is accomplished correctly the integration of the differential equation with a complex g-function leads to non-closed form solutions for stress. In addition, the current values of strain hardening rate must be always updated during numerical integration. Therefore the CPU may be much extended in more complicated engineering problems. In conclusion, the RK

Table 1
Classification of the processes as a function of the strain rates in terms of inertia and thermal coupling

Applications	$\dot{\varepsilon}_p (s^{-1})$	Inertia	Thermal processes
Static	$<10^{-4}\ s^{-1}$		
Punching, cutting ...	$10^{-3}\ s^{-1}$–$10^{2}\ s^{-1}$	Negligible	Isothermal
Crash test	$1\ s^{-1}$–$5*10^{2}\ s^{-1}$	Important	Adiabatic
High speed machining, ballistic, impact, ring expansion...	Up to $10^{3}\ s^{-1}$		