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Dimensionless parameters in symmetric double lap joints: An orthotropic solution for thermomechanical loading

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Abstract

Two thermomechanical analytical models are developed for orthotropic double lap joints with a view to identifying key dimensionless parameters that describe the behavior of the joint under combined thermal–mechanical loads. The solutions, based on the principle of virtual work, differ in the complexity of the assumed stress field. The first solution is similar to Volkersen [Volkersen, O., 1938. Die niektraftverteilung in zugbeanspruchten mit konstanten laschenquerschritten. Luftfahrtforschung 15, 41–47] with the addition of orthotropic and thermal effects. The second solution, extending the work of Davies [Davies, G.A.O., 1982. Virtual Work in Structural Analysis, John Wiley & Sons, New York] captures the peel stress as well as the traction free boundary condition at the adhesive edge. Relevant non-dimensional parameters are identified in terms of geometric, material, and load quantities. A dimensionless load ratio is identified which dictates the shape of the stress distribution. This ratio can also be used to quickly determine the dominant loading mechanism. Dimensionless stress plots are presented for representative lap joints.

Keywords: Lap joint; Adhesive bond; High temperature; Composite; Thermal expansion; Non-dimensional parameters

1. Introduction

The use of composite materials continues to increase in the aerospace industry, which places an increasing importance on the ability of designers to properly specify the performance of bonded structural joints. Due to specific strength, specific stiffness, and efficient load distribution and load transfer, recent high profile aircraft and spacecraft have featured bonded joints. New epoxies and adhesives have shown great promise to expand the temperature range over which structural fiber reinforced polymer composites are used. These materials provide an opportunity to replace specialized, non-structural thermal protection with integrated composite systems capable of carrying structural load over a range of temperature extremes. Consequently, temperature resistant composite structures and bonded joints will be used in increasing quantities. In addition to the harsh operating environments, the processing temperatures for these specialized epoxies and adhesives are also quite

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Nomenclature

t_κ	material thicknesses of component κ (m)
l	lap length (m)
x	lap coordinate measured from the left edge (m)
y	lap coordinate measured from the lower edge (m)
$\sigma_{\kappa 11(x)}$	longitudinal stress in component κ (Pa)
$\sigma_{\kappa 22(x,y)}$	transverse stress in component κ (Pa)
$\tau_{\kappa 12(x)}$	shear stress in component κ (Pa)
$E_{\kappa ii}$	orthotropic engineering moduli of component κ (Pa)
G_{b12}	shear modulus of the adhesive (Pa)
$E_{p[0l]}$	Young's moduli of the end posts (Pa)
$\nu_{\kappa ij}$	Poisson's ratios of component κ
$\alpha_{\kappa ii}$	orthotropic thermal expansion coefficient of component κ (°C^{-1})
P	mechanical load applied to joint, per unit depth (N m^{-1})
ΔT	temperature change from reference temperature (°C)
F	mechanical load carried by an end post (N)
c_0, d_0	coefficients of assumed stress distribution (N)
c_1, d_1	coefficients of assumed stress distribution (N m^{-1})
ψ_P	mechanical load parameter (N m^{-4})
ϕ_P	mechanical load parameter (N m^{-6})
ψ_T	thermal load parameter (N m^{-4})
ϕ_T	thermal load parameter (N m^{-6})
ω	system parameter (m^{-1})
β	system parameter (m^{-2})
γ	system parameter (m^{-4})
$\bar{x}$	dimensionless coordinate $\frac{x}{l}$ measured from the left edge of the adhesive
$\overline{\omega}$	dimensionless system parameter
$\overline{\beta}$, $\overline{\gamma}$	dimensionless system parameters
$\overline{\lambda}_1$, $\overline{\lambda}_3$	dimensionless system parameters
$\overline{\sigma}_{\kappa 11(x)}$	dimensionless longitudinal stress in component κ
$\overline{\sigma}_{\kappa 22(x,y)}$	dimensionless transverse stress in component κ
$\overline{\tau}_{\kappa 12(x)}$	dimensionless shear stress in component κ
$\overline{\overline{\sigma}}_{a11(x)}$	normalized dimensionless longitudinal stress in component a
$\overline{\psi}_P$, $\overline{\phi}_P$	dimensionless mechanical load parameters
$\overline{\psi}_T$, $\overline{\phi}_T$	dimensionless thermal load parameters
$\overline{\phi}_{aR}$, $\overline{\phi}_{cR}$	dimensionless thermal to mechanical load ratios
$\overline{\phi}_{\text{total}}$	dimensionless total load parameter
$\overline{\overline{\phi}}_P$	dimensionless mechanical load fraction
$\bar{a}, \bar{b}, \bar{A}, \bar{B}, \bar{C}, \bar{D}$	dimensionless coefficients
$\bar{\bar{a}}, \bar{\bar{b}}, \bar{\bar{A}}, \bar{\bar{B}}, \bar{\bar{C}}, \bar{\bar{D}}$	dimensionless coefficients
$\hat{\sigma}_{\kappa 11(x)}$	longitudinal virtual stress in component κ
$\hat{\sigma}_{\kappa 22(x,y)}$	transverse virtual stress in component κ
$\hat{\tau}_{\kappa 12(x)}$	shear virtual stress in component κ
[]	the *or* operator, i.e. [13] is 1 *or* 3 (no sum)
κ	$\kappa = [abc]$ (no sum) representing central adherend (a), adhesive (b), and outer adherend (c), respectively
ii	$i = [123]$ (no sum)
ij	$i,j = [123]$ where $i \neq j$ (no sum)