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Destabilization of long-wavelength Love and Stoneley waves in slow sliding

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ABSTRACT

Love waves are dispersive interfacial waves that are a mode of response for anti-plane motions of an elastic layer bonded to an elastic half-space. Similarly, Stoneley waves are interfacial waves in bonded contact of dissimilar elastic half-spaces, when the displacements are in the plane of the solids. It is shown that in slow sliding, long-wavelength Love and Stoneley waves are destabilized by friction. Friction is assumed to have a positive instantaneous logarithmic dependence on slip rate and a logarithmic rate weakening behavior at steady-state.

Long-wavelength instabilities occur generically in sliding with rate- and state-dependent friction, even when an interfacial wave does not exist. For slip at low rates, such instabilities are quasi-static in nature, i.e., the phase velocity is negligibly small in comparison to a shear wave speed. The existence of an interfacial wave in bonded contact permits an instability to propagate with a speed of the order of a shear wave speed even in slow sliding, indicating that the quasi-static approximation is not valid in such problems.

1. Introduction

Destabilization of interfacial elastic waves due to friction has been a topic of some recent investigations (Adams, 1995; Ranjith and Rice, 2001). For in-plane elasticity problems, where displacements are confined to the plane of the solids, two well-known interfacial waves are the Stoneley wave (Stoneley, 1924) and the slip wave (Achenbach and Epstein, 1967). The Stoneley wave occurs in bonded contact of dissimilar elastic half-spaces while the slip wave, also called the generalized Rayleigh wave, is for a freely slipping interface between two half-spaces. There are no analogues of the Stoneley wave and the slip wave in anti-plane elasticity, where the displacement is normal to the plane of the solids. However, an interface wave solution does exist in the bonded contact of a *finite* layer on a half-space. This is the Love wave (Love, 1911). The Love wave differs from the Stoneley and slip waves in that (a) it always exists if the shear wave speed of the layer is greater than that of the half-space whereas the other two interfacial waves exist only when the shear wave speeds of the solids are not very different (b) its speed along the interface is greater than the shear wave speed of the layer but less than that of the substrate, while the other two waves are subsonic (c) it is dispersive and the dispersion relations are multi-valued.

In this paper, two problems are studied involving dissimilar materials that permit interfacial waves in bonded contact. Anti-plane sliding of a finite layer on an elastic half-space is first studied. In slow frictional sliding, it is shown that the Love wave is destabilized at long wavelengths. In-plane sliding of dissimilar elastic half-spaces is subsequently analyzed. It is shown that long-wavelength Stoneley waves are also destabilized in slow sliding.

2. The anti-plane problem

In this section, the equation governing the stability of steady sliding of an elastic layer on an elastic half-space is derived. The perturbations from steady sliding are assumed to be transverse to the direction of slip (i.e., anti-plane sliding). The elastodynamic relation between slip and shear stress perturbations is first derived. A friction law which also relates the slip and shear stress perturbations is then introduced. These two relations are used to obtained the equation governing slip stability.

Consider an isotropic elastic layer of thickness h sliding on an isotropic elastic half-space with a steady rate V_o (Fig. 1). The steady motion is due to an applied shear stress τ_o which is at the friction threshold, $\tau_o = f\sigma_o$, where σ_o is the compressive normal stress on the boundary of the layer and f is the friction coefficient at slip rate V_o. The shear modulus, density and shear wave speed of the layer are denoted by μ, ρ and c_s, respectively, and corresponding properties of the half-space are denoted by μ', ρ' and c'_s.

A Cartesian coordinate system is located as shown in Fig. 1 so that the interface between the solids is at $x_2 = 0$ and the layer slides in the x_3 direction. The elastic fields are assumed to be independent of the x_3 coordinate. We are interested in the relation between slip and stress perturbations at the interface when the perturbation is

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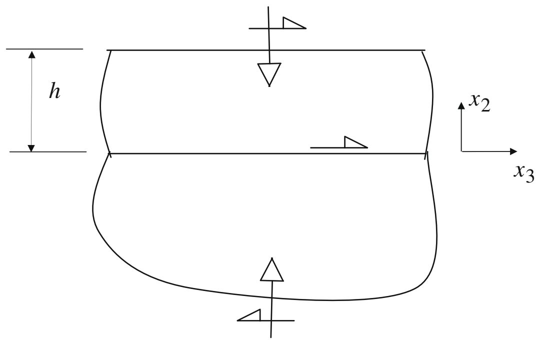

Fig. 1. Geometry for the anti-plane sliding problem.

transverse to the direction of slip, namely in the x_1 direction. If $u_i(x_1, x_2, t)$, $i = 1,2,3$ denote the displacement field, due to isotropy of the solid, the only displacement component is that in the direction of slip:

$$u_1 = u_2 = 0,\\ u_3 = u_3(x_1, x_2, t). \tag{1}$$

Let $\tau_{ij}(x_1, x_2, t), i, j = 1,2,3$ denote the stresses. The only non-zero stresses corresponding to the displacement field Eq. (1) are $\tau_{13} = \tau_{31}$ and $\tau_{23} = \tau_{32}$. They are given by

$$\tau_{13} = \mu \frac{\partial u_3}{\partial x_1},\\ \tau_{23} = \mu \frac{\partial u_3}{\partial x_2}, \tag{2}$$

the latter being the traction component on planes normal to the x_2 direction.

For the layer, the equation of motion in terms of the stresses is

$$\frac{\partial \tau_{13}}{\partial x_1} + \frac{\partial \tau_{23}}{\partial x_2} = \rho \frac{\partial^2 u_3}{\partial t^2}. \tag{3}$$

Substituting for the stresses from Eq. (2), one gets

$$\frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_2^2} = \frac{1}{c_s^2}\frac{\partial^2 u_3}{\partial t^2}, \tag{4}$$

where $c_s = \sqrt{\mu/\rho}$. Similarly, the equation of motion of the elastic half-space in the region $x_2 < 0$ is

$$\frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_2^2} = \frac{1}{c_s'^2}\frac{\partial^2 u_3}{\partial t^2}. \tag{5}$$

where $c_s' = \sqrt{\mu'/\rho'}$ is the shear wave speed of the half-space.

Slip at rate V_o and a perturbation from it in a single Fourier mode of wavenumber k can be represented by a displacement field of the form

$$u_3(x_1, x_2, t) = V_o t + U^+(k,p)e^{ikx}e^{\alpha x_2}e^{pt}, \quad x_2 > 0,\\ u_3(x_1, x_2, t) = U^-(k,p)e^{ikx_1}e^{\alpha' x_2}e^{pt}, \quad x_2 < 0. \tag{6}$$

where p is a complex variable, dependent on k, which characterizes the time response to the perturbation. $\alpha(k,p)$ and $\alpha'(k,p)$ are to be determined so that the governing equations of motion are satisfied. Substituting into the equation of motion for the layer, Eq. (4), gives

$$\alpha^2 = k^2 + \frac{p^2}{c_s^2}. \tag{7}$$

Defining

$$\alpha = |k|\sqrt{1 + p^2/k^2 c_s^2}, \tag{8}$$

where $\sqrt{}$ denotes the analytic continuation of the positive square root function, both α and $-\alpha$ solve Eq. (7). A convenient choice of branch cuts in the complex p-plane is from the branch points $p = \pm i\ |k|\ c_s$ to $p = \infty$ along the imaginary axis, away from the origin. The general form of the displacement in the layer is therefore

$$u_3(x_1, x_2 > 0, t) = V_o t + \left[U_1^+(k,p)e^{-\alpha x_2} + U_2^+(k,p)e^{\alpha x_2}\right]e^{ikx_1}e^{pt} \tag{9}$$

The stress component τ_{23} in the layer corresponding to the above displacement field is

$$\tau_{23}(x_1, x_2 > 0, t) = \tau_o + \mu\left[-\alpha U_1^+(k,p)e^{-\alpha x_2} + \alpha U_2^+(k,p)e^{\alpha x_2}\right]e^{ikx_1}e^{pt} \tag{10}$$

The perturbations at the interface do not alter the applied shear stress τ_o on the boundary of the layer. Thus $\tau_{23}(x_1, h, t) = \tau_o$, so that

$$-U_1^+ e^{-\alpha h} + U_2^+ e^{\alpha h} = 0. \tag{11}$$

An analogous development for the half-space $x_2 < 0$ follows. The displacement field in the half-space is of the form

$$u_3(x_1, x_2 < 0, t) = U^-(k,p)e^{ikx_1}e^{\alpha' x_2}e^{pt}. \tag{12}$$

Substituting into the equation of motion for the half-space gives

$$\alpha'^2 = k^2 + \frac{p^2}{c_s'^2}, \tag{13}$$

which has the solution

$$\alpha' = |k|\sqrt{1 + p^2/k^2 c_s'^2}. \tag{14}$$

Branch cuts are defined as before from $p = \pm i|k|c_s'$ to $p = \infty$ along the imaginary axis, away from the origin. This ensures that $\mathrm{Re}(\alpha') \geqslant 0$ for any p. It is noted that $-\alpha'$ is not a valid solution to Eq. (13) since it gives rise to an unbounded displacement field as $x_2 \to -\infty$.

The stress component τ_{23} in the half-space is then

$$\tau_{23}(x_1, x_2 < 0, t) = \tau_o + \mu'\alpha' U^-(k,p)e^{\alpha' x_2}e^{ikx_1}e^{pt}. \tag{15}$$

The slip at the interface is

$$\delta(x_1, t) = u_3(x_1, x_2 = 0^+, t) - u_3(x_1, x_2 = 0^-, t)\\ = V_o t + [U_1^+ + U_2^+ - U^-]e^{ikx_1}e^{pt}. \tag{16}$$

Denoting

$$D(k,p) \equiv U_1^+(k,p) + U_2^+(k,p) - U^-(k,p), \tag{17}$$

the slip can be written as

$$\delta(x_1, t) = V_o t + D(k,p)e^{ikx_1}e^{pt}. \tag{18}$$

The traction component of stress at the interface

$$\tau(x_1, t) = \tau_{23}(x_1, 0, t) \equiv \tau_o + T(k,p)e^{ikx_1}e^{pt} \tag{19}$$

is continuous. From Eqs. (10) and (15), this requires

$$-\mu\alpha U_1^+ + \mu\alpha U_2^+ = \mu'\alpha' U^-. \tag{20}$$

Eqs. (11), (17) and (20) constitute a system of linear algebraic equations for U_1^+, U_2^+ and U^- in terms of D. Solving that system,

$$U^- = -\frac{\mu\alpha}{\mu\alpha + \mu'\alpha' \coth \alpha h} D. \tag{21}$$

The shear stress at the interface is then

$$\tau(x_1, t) = \tau_o - \frac{\mu'\alpha'\mu\alpha}{\mu\alpha + \mu'\alpha' \coth \alpha h} D(k,p)e^{ikx_1}e^{pt}. \tag{22}$$

The amplitudes of the shear stress and slip perturbations at the interface thus satisfy