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Analysis of multiple off-axis ply cracks in composite laminates

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ABSTRACT

This paper presents a synergistic methodology to analyze damage behavior in composite laminates with transverse matrix cracks in plies of multiple orientations. The approach combines the strengths of micro-damage mechanics (MDM) and continuum damage mechanics (CDM) in predicting the stiffness degradation due to presence of transverse cracks. The micromechanics is performed on a representative unit cell using a three-dimensional finite element analysis to calculate the crack opening displacement (COD) accounting for the influence of the surrounding plies, the so-called constraint effect. This information is then incorporated in the CDM formulation dealing with laminates containing cracks in different ply orientations through a 'constraint parameter'. In CDM, a separate damage mode is defined for each type of crack and the expressions for engineering moduli of the damaged laminate are derived in terms of crack density and the constraint parameter. The COD and stiffness degradation predictions agree well with published experimental data for $[0/\pm\theta_4/0_{1/2}]_s$ laminate configuration. To enable damage analysis of other configurations of $[0_m/\pm\theta_n/0_{m/2}]_s$ laminate, a parametric study of the CODs is performed and using the computations a master equation is developed.

1. Introduction

In an un-constrained uni-directional composite, such as a ply not bonded to other plies, a uniform tensile stress applied normal to fibers will cause failure from a single crack lying in the matrix between fibers or at the fiber/matrix interface. However, if the composite is constrained, such as a ply within a laminate, then failure does not result from a single crack. Instead, multiple cracks form as the applied load increases. This phenomenon is described as multiple matrix cracking (Talreja, 2006). These matrix cracks usually form first in the ply thickness direction and then grow along fibers spanning the laminate width. Matrix cracks, usually the first mode of damage, are not critical from a final failure point of view but can lead to a significant reduction in individual ply properties as well as in the laminate properties. Modeling of stiffness degradation subsequent to matrix damage has been the topic of extensive research in the recent decades, especially for cross-ply laminates ($[0_m/90_n]_s$). A variety of analytical approaches have been suggested, e.g., ply-discount method, shear lag models (Highsmith and Reifsnider, 1982; Lim and Hong, 1989), variational method (Hashin, 1985), self-consistent approximation (Dvorak et al., 1985), 3-D laminate theory (Gudmundson and Ostlund, 1992) and continuum damage mechanics (Talreja, 1985a; Allen et al., 1987). However, most of the research work is limited to cross-ply laminates, which are easier to analyze but are not used often in practical applications. Analyzing damage in laminates of general layup is quite challenging due to multiplicity of damage modes and the constraints induced on individual ply cracks by the neighboring plies.

Composite laminates with off-axis plies are important for applications where structures undergo loading combinations that necessitate use of multiple fiber orientations to generate required properties. Still, damage in such laminates has not been fully analyzed. Masters and Reifsnider (1982) experimentally observed damage development in quasi-isotropic ($[0/\pm45/90]_s$ and

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$[0/90/\pm 45]_s$) carbon–epoxy laminates under fatigue loading. Tong et al. (1997a) conducted experimental investigations of matrix crack growth behavior under quasi-static and fatigue loadings in quasi-isotropic glass–epoxy laminates and used generalized plane-strain finite element analysis to predict stiffness degradation and ply stress distribution (Tong et al., 1997b). However, damage in off-axis plies is essentially a 3-D stress analysis problem and a generalized plane strain formulation cannot adequately address it. Other approaches such as equivalent constraint model (ECM), in combination with the first order shear deformation laminated plate theory (FSDT) (Zhang and Herrmann, 1999) and with a modified shear lag analysis (Kashtalyan and Soutis, 2000), have also been attempted. Recently, Yokozeki and Aoki (2004, 2005a,b) have analyzed laminates with obliquely crossed matrix cracks utilizing a two-dimensional shear lag analysis. Considering the complexity of the problem at hand, these works are good starting point but they provide approximate solutions whose accuracy cannot be fully verified as no exact analytical solutions currently exist.

To analyze the deformational response of composite laminates subsequent to matrix cracking, the most direct measure of crack influence over laminate properties is the "coefficient" of crack opening displacement (COD), i.e., average crack surface separation per unit of an applied load quantity. Only a few researchers in the past have focused on surface displacements of ply cracks in an explicit manner. Gudmundson and Ostlund (1992) derived analytical expressions for average stiffness properties of cracked symmetric laminates in terms of COD. They assumed that the average COD for matrix cracks in a composite laminate of general layup could be approximated by the analytical solutions of an array of parallel cracks in an infinite homogeneous medium. However, this completely neglects the constraint effect on COD and evaluating the consequence of such approximation is therefore necessary either by experimental or computational means. To study COD, Varna et al. (1993) developed a device to experimentally measure COD in cross-ply laminates. A separate study (Varna et al., 1997) verified the accuracy of different analytical models for estimating COD in cross-ply laminates. Joffe et al. (2001) later used a FE plane stress model to evaluate the average COD dependence on the crack spacing and on the constraint of adjacent sub-laminates.

The classical CDM approach is quite efficient in predicting stiffness degradation if certain phenomenological constants can be evaluated (Talreja, 1985b). An experimental evaluation of the constants may not be easy in all cases and to alleviate this limitation, Talreja (1996) later proposed a synergistic damage mechanics (SDM) approach and illustrated it to describe the deformational response of $[\pm\theta/90_2]_s$ laminates. This approach combines micromechanics and continuum damage mechanics judiciously to produce a versatile methodology. The micromechanical damage mechanics, or briefly, micro-damage mechanics (MDM) performs analysis of local stress-redistributions due to cracking, incorporating the micro-level geometry. On the other hand, CDM, as formulated by Talreja (1985a, 1991), allows a specific output of MDM (COD) to be used within a representative volume element (RVE), i.e., at meso level. In this way, the synergism between micromechanics and CDM effectively treats the multi-scale nature of damage. More recently, the SDM approach has also been extended to analyze viscoelastic behavior of composites with damage (Varna et al., 2004).

In the present study, we present a synergistic methodology to deal with matrix cracks in plies with multiple off-axis orientations. The continuum damage mechanics formulation of particular relevance to this work is the one presented by Talreja (1991, 1994), wherein the damage state in the laminate is described by second order tensors. In the present scenario, the damage state can be suitably represented by damage mode tensors regarding ply cracking in each orientation as an individual damage mode. The constraint effects on the cracked plies imposed by the surrounding plies are evaluated in terms of the COD changes in off-axis plies. CODs are determined using 3-D FE analysis and then subsequently used in the CDM model through the constraint parameter to predict the stiffness properties of the degraded laminate. The CODs are evaluated at different applied strains and the variation in these is checked against the experimental values at 0.4% and 0.6% strains. The strength of SDM approach lies in accurate COD calculation using computational micromechanics and an accurate damage description using CDM. Thus, SDM promises to be a pragmatic solution to the damage analysis problem for laminates with complex layups for which analytical results are difficult to derive. In order to make the SDM approach more versatile, detailed parametric studies are conducted to study the variables which may affect the constraint of un-cracked plies over cracked plies. Using these parametric studies, a master equation for CODs in terms of geometry and material variables is proposed. Also, the profile of average crack surface displacements through thickness of the cracked ply is studied for different laminate configurations. Finally, stiffness moduli for $[0_m/\pm\theta_n/0_{m/2}]_s$ laminates are predicted for different stiffness and thicknesses of cracked and supporting plies.

2. Synergistic damage mechanics methodology for $[\mathbf{0}_m/\pm\theta_n/\mathbf{0}_{m/2}]_s$ laminates

Consider a symmetric laminate with a general layup, e.g. $[0_m/\pm\theta_n/\phi_p]_s$, loaded in axial tension, with ϕ restricted to angles which do not cause cracking. The loading will produce an in-plane stress state in each off-axis ply consisting of normal stresses along and perpendicular to fibers in that ply and a shear stress in the plane of the ply. Depending on the values of θ, ϕ and ply properties, the stress perpendicular to the fibers could be tensile or compressive. Thus, on loading, an off-axis ply may or may not develop intralaminar cracks. When $\theta = 90^\circ$ the matrix will undergo multiple cracking in the transverse plies. For other cases of off-axis ply orientations, multiple cracking is typically observed to occur for angles from 50° to 90°. However, it has been observed that even in cases where these cracks do not initiate in the off-axis plies, the laminate moduli change with the applied load due to shear stress induced damage within the plies (Varna et al., 1999b).

When cracks are formed, the opening and sliding of crack surfaces alter the stress and strain states in the cracked plies, thereby changing the global deformational response of the laminate. The damage state in the laminate representative