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Shakedown of a cohesive-frictional half-space subjected to rolling and sliding contact

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Abstract

The problem of rolling and sliding contact of a cylinder on the surface of a half-space of cohesive-frictional material is considered. Three shakedown multipliers, of which two are upper bounds and one is exact are computed using a simple numerical procedure. This latter solution differs significantly from previously published analytical solutions which, for realistic material parameters, typically overestimate the shakedown load by a factor of 1.5–2.5.

Keywords: Shakedown; Rolling; Hertz contact; Pavement design

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1. Introduction

The shakedown limit has since the 1960's been recognized as the rational design criterion for metallic contacts such as rails, roller bearings, and traction drives (Johnson, 1987). More recently, starting with the work of Sharp and Booker (1984), it has been suggested that the shakedown limit would be an equivalently rational design criterion for road pavements subjected to traffic loads. Although the available experimental evidence is not entirely conclusive, it would seem that shakedown theory does offer important insights into the mechanics of the progressive degradation of road pavements (Sharp and Booker, 1984; Raad et al., 1988, 1989, 2005; Radovsky and Murashina, 1996; Shiau, 2001). Recently, shakedown concepts have also been used to describe the ratcheting behaviour of granular materials in general (Garcia-Rojo and Herrmann, 2005; Garcia-Rojo et al., 2005).

The application of shakedown theory to the geomaterials that usually make up the pavement subgrade requires consideration of a general cohesive-frictional yield criterion (as opposed to the purely cohesive criteria used in metal plasticity). This feature significantly complicates the determination of the shakedown limit, both in numerical and in analytical computations. In the past a number of attempts of both types of calculations have been made. Firstly, in their pioneering paper, Sharp and Booker (1984) introduced the so-called method

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of conics, which can be seen as an application of Melan's lower bound theorem, and solved a number of two-dimensional problems where the load was approximated in terms of a trapezoidal pressure distribution. This method was later given a kinematic interpretation by Collins and Cliffe (1987) and the resulting generalized upper bound method was applied to a number of more realistic three-dimensional problems (Collins et al., 1993; Collins and Boulbibane, 2000; Boulbibane et al., 2005).

Secondly, the problem has been treated numerically using a combination of finite elements and linear and nonlinear programming techniques (Yu and Hossain, 1998; Shiau, 2001; Boulbibane and Ponter, 2005; Li and Yu, 2006). Although these procedures are often advocated as being either upper or lower bound procedures, they can in fact only be seen as being approximate since rigorous application of the shakedown theorems implies that one knows the exact elastic stresses at every point of the domain. Furthermore, due to the lack of exact shakedown solutions, these procedures are generally difficult to validate and one can observe a large scatter in the results for even the simplest problems (Krabbenhoft et al., 2006).

Recently, Yu (2005) has sought to improve this unsatisfactory state of affairs by providing an analytical solution for two classical benchmark problems, namely those of a homogeneous half-space subjected to either two or three-dimensional Hertzian contact. This constitutes an important contribution towards establishing exact solutions by which numerical procedures can be validated. The solutions of Yu (2005) were derived by using the lower bound theorem of elastic shakedown. For cases with subsurface failure, Yu's solutions are shown to give rigorous lower bounds.

When surface failure becomes critical, however, Yu's solutions may exceed those from a rigorous lower bound analysis due to the fact that the residual stresses are not constrained to satisfy the yield condition and equilibrium (unlike the solutions of Yu and Hossain (1998)). In this paper we employ the general approach of Yu (2005), amended to include the effects of the residual stress yield condition and residual stress equilibrium, to derive shakedown results for the problem of two-dimensional Hertzian contact. The full procedure is described in detail, and rigorous results for the elastic shakedown load are obtained for all values of the surface-roller friction coefficient.

2. Problem definition

The general problem of a pavement subjected to traffic load can, as a first approximation, be idealized in terms of a homogeneous half-space subjected to the action of an infinitely long roller (Radovsky and Murashina, 1996; Yu and Hossain, 1998). As such the problem may be modeled as one of 2D plane strain elasticity/plasticity as sketched in Fig. 1.

2.1. Contact pressures

The pressure distribution due to the roller is modeled as a Hertzian contact with vertical and horizontal components given by

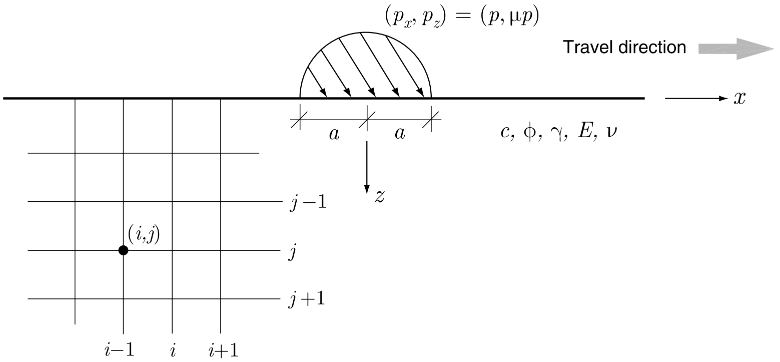

Fig. 1. Half-space of cohesive-frictional material subjected to rolling and sliding contact.