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# Scattering of a longitudinal wave by a circular crack in a fluid-saturated porous medium

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## Abstract

Physical properties of many natural and man-made materials can be modelled using the concept of poroelasticity. Some porous materials, in addition to the network of pores, contain larger inhomogeneities such as inclusions, cavities, fractures or cracks. A common method of detecting such inhomogeneities is based on the use of elastic wave scattering. We consider interaction of a normally incident time-harmonic longitudinal plane wave with a circular crack imbedded in a porous medium governed by Biot's equations of dynamic poroelasticity. The problem is formulated in cylindrical co-ordinates as a system of dual integral equations for the Hankel transform of the wave field, which is then reduced to a single Fredholm integral equation of the second kind. It is found that the scattering that takes place is predominantly due to wave induced fluid flow between the pores and the crack. The scattering magnitude depends on the size of the crack relative to the slow wave wavelength and has it's maximum value when they are of the same order.


*Keywords:* Poroelasticity; Biot theory; Slow wave; Fractured medium

## 1. Introduction

Physical properties of many natural and man-made materials, such as rocks, soils, foams, biological tissues and construction materials, can be modelled using the concept of poroelasticity. A poroelastic material consists of an elastic frame permeated by an interconnected pore space filled with a Newtonian fluid. Many porous materials, in addition to the network of pores, contain larger inhomogeneities such as inclusions, cavities, fractures or cracks. A common method of detecting such inhomogeneities is based on the use of elastic wave scattering. This method is widely used in such diverse applications as non-destructive testing and oil exploration. The effect of a distribution of inhomogeneities on the passing wave can be estimated using multiple-scattering theory. This approach requires knowledge of the scattering that takes place due to the presence of a single inhomogeneity, which is the subject of this paper.

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We consider the interaction of a plane longitudinal wave in a fluid-saturated porous medium with an open oblate spheroidal crack of radius $a$ and small thickness $2c \ll a$ placed perpendicular to the direction of wave propagation. The background porous medium is assumed to be governed by Biot's equations of dynamic poroelasticity (Biot, 1962). This poroelastic problem is analogous to the problem of scattering by a crack in an elastic medium (Robertson, 1967; Garbin and Knopoff, 1973; Piau, 1979). In particular, Robertson (1967) formulated this problem in cylindrical co-ordinates as a system of dual integral equations in the Hankel transform of the wave field, which was then reduced to a Fredholm equation of the second kind. For fluid-saturated porous materials the interaction differs from that of the corresponding elastic scattering, as it involves flow of the pore fluid between the crack and the host medium, induced by the passing wave. This effect is particularly significant for thin cracks, as their high compliance (compared to that of the relatively stiff pores) causes the fluid to flow in and out of the crack during rarefaction and compression wave cycles. Recently, similar problems have been investigated in the fields of poroelasticity and the mathematically analogous thermoelasticity. Jin and Zhong (2002) investigated the dynamic stress intensity factor of a circular crack in an infinite poroelastic solid, however they only treated the case of impermeable crack surfaces. Sherief and El-Maghraby (2003) solve a dynamical problem for an infinite thermoelastic solid with an internal circular crack which is subjected to prescribed temperature and stress distributions. We apply a similar approach to the scattering by a crack in a porous medium.

We restrict the analysis to frequencies that are small compared to Biot's characteristic frequency. For typical porous materials such as reservoir rocks and soils this assumption is appropriate for frequencies of up to 10–100 kHz.

## 2. Problem formulation

### 2.1. Equations of poroelasticity

We consider an incident plane longitudinal wave, harmonic in time, propagating in a fluid-saturated porous medium in the positive direction of the $z$-axis of a cylindrical co-ordinate system. This wave can be represented as the displacement field $u_z^{(i)} = u_0 e^{ik_1 z}$, where $k_1$ is the wavenumber. We aim to derive the scattered field $\mathbf{u}(\mathbf{r})$ that results from interaction between the incident wave and the crack, which occupies the circle $0 \leqslant r \leqslant a$ in the plane $z = 0$. The total displacement field is therefore $\mathbf{u}^{(t)}(\mathbf{r}) = u_z^{(i)}\mathbf{a}_z + \mathbf{u}(\mathbf{r})$, where $\mathbf{a}_z$ is a unit vector directed along the $z$-axis.

Since we have geometrical symmetry about the crack plane $z = 0$, both the scattered and total displacement fields satisfy the following equations of dynamic poroelasticity (Biot, 1962) in the semi-infinite poroelastic medium $z \geqslant 0$:

$$\nabla \cdot \boldsymbol{\sigma} = -\omega^2(\rho\mathbf{u} + \rho_f \mathbf{w}), \tag{1}$$

$$\nabla p = \omega^2(\rho_f \mathbf{u} + q\mathbf{w}), \tag{2}$$

where $\mathbf{w} = \phi(\mathbf{U} - \mathbf{u})$ is the relative fluid displacement, $\phi$ is the medium porosity, $\mathbf{U}$ is the average absolute fluid displacement, $\omega$ is the angular wave frequency, $\rho_f$ and $\rho$ are the densities of the fluid and of the overall medium. At low frequencies $q(\omega) \approx i\eta/\kappa\omega$ (Biot, 1956) is a frequency-dependent coefficient responsible for viscous and inertial coupling between the solid and fluid displacements where $\eta$ is the fluid viscosity and $\kappa$ is the intrinsic permeability of the medium. $\boldsymbol{\sigma}$ and $p$ are the total stress tensor and fluid pressure, which are related to the displacement vectors via the constitutive relations

$$\boldsymbol{\sigma} = [(H - 2\mu)\nabla \cdot \mathbf{u} + \alpha M \nabla \cdot \mathbf{w}]\mathbf{I} + \mu[\nabla \mathbf{u} + (\nabla \mathbf{u})^T], \tag{3}$$

$$p = -\alpha M \nabla \cdot \mathbf{u} - M \nabla \cdot \mathbf{w}. \tag{4}$$

In Eqs. (3) and (4) $\mu$ is the shear modulus of the solid frame, $\alpha = 1 - K/K_g$ is the Biot–Willis coefficient (Biot and Willis, 1957),

$$M = \left[\frac{(\alpha - \phi)}{K_g} + \frac{\phi}{K_f}\right]^{-1} \tag{5}$$

is the so-called pore space modulus,