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A variational asymptotic micromechanics model for predicting thermoelastic properties of heterogeneous materials

Wenbin Yu *, Tian Tang

Department of Mechanical and Aerospace Engineering, Utah State University, Logan, UT 80322-4130, USA

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Abstract

A variational asymptotic micromechanics model has been developed for predicting effective thermoelastic properties of composite materials, and recover the local fields within the unit cell. This theory adopts essential assumptions within the concept of micromechanics, achieves an excellent accuracy, and provides a unified treatment for 1D, 2D, and 3D unit cells. This theory is implemented using the finite element method into the computer program, VAMUCH, a general-purpose micromechanics analysis code. Several examples are used to validate the theory and the code. The results are compared with those available in the literature and those produced by a commercial finite element package.

Keywords: Heterogeneous; Thermoelastic; Micromechanics; Homogenization; Variational asymptotic method

1. Introduction

In recent years, more and more structures are made of composite materials with engineered microstructure for better performance. Because the macroscopic structural dimensions are usually several orders of magnitude larger than the characteristic size of constituents, it is not very practical to analyze such structures by meshing all the details of constituent materials. Usually, the concept of unit cell (UC) is used to create a pseudo, ‘‘effective’’, material with homogeneous properties from the original heterogeneous materials with so-called micromechanics models; see Fig. 1.

In the past several decades, numerous micromechanics models have been developed; see Hashin (1983) and references cited therein. The simplest models are the rules of mixtures based on Voigt and Reuss hypotheses, which provide the upper and lower bounds, respectively (Hill, 1952). The difference between these two bounds could be too large to be useful for general composite materials. To this end, researchers have proposed various techniques to either reduce the difference between the bounds, or find an approximate value between the upper and lower bounds. Typical approaches are the self-consistent model (Hill, 1965) and its

* Corresponding author. Tel: +1 435 7978246; fax: +1 435 7972417.
E-mail address: wenbin.yu@usu.edu (W. Yu).

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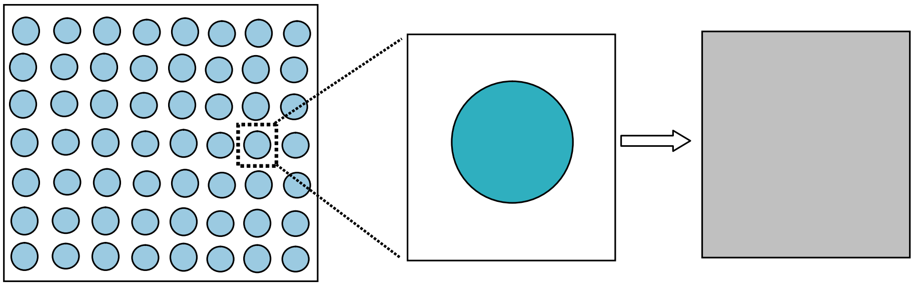

Fig. 1. Heterogenous material, unit cell, and effective material.

generalizations (Dvorak and Bahei-El-Din, 1979; Accorsi and Nemat-Nasser, 1986), the variational approach of Hashin and Shtrikman (1962), third-order bounds (Milton, 1981), the method of cells (MOC) (Aboudi, 1982) and its variants (Aboudi, 1989; Paley and Aboudi, 1992; Aboudi et al., 2001; Williams, 2005), recursive cell method (Banerjee and Adams, 2004), mathematical homogenization theories (MHT) (Bensoussan et al., 1978; Murakami and Toledano, 1990), finite element approaches using conventional stress analysis of a representative volume element (RVE) (Sun and Vaidya, 1996), and many others. Although these approaches were originally introduced to predict elastic properties, most of these approaches are also extended to predict thermomechanical properties; see Schapery (1968), Rosen and Hashin (1970), and Aboudi (1984) for a few examples.

Recently, a new technique for micromechanics modeling, namely variational asymptotic method for unit cell homogenization (VAMUCH) (Yu and Tang, 2007), has been developed based on the variational asymptotic method of Berdichevsky (1979). The technique invokes two essential assumptions within the concept of micromechanics:

- *Assumption* 1. The exact solutions of the field variables have volume averages over the UC. For example, if u_i are the exact displacements within the UC, there exist v_i such that

$$v_i = \frac{1}{\Omega} \int_{\Omega} u_i \, \mathrm{d}\Omega \equiv \langle u_i \rangle \tag{1}$$

where Ω denotes the domain occupied by the UC and its volume.

- *Assumption* 2. The effective material properties obtained from the micromechanical analysis of the UC are independent of the geometry, the boundary conditions, and loading conditions of the macroscopic structure, which means that effective material properties are assumed to be the intrinsic properties of the material when viewed macroscopically.

Note that these assumptions are not restrictive. The mathematical meaning of the first assumption is that the exact solutions of the field variables are integrable over the domain of UC, which is true almost all the time. The second assumption implies that we can neglect the size effects of the material properties in the macroscopic analysis, which is an assumption often made in the conventional continuum mechanics. Of course, the micromechanical analysis of the UC is only needed and appropriate if $\eta = h/l \ll 1$, with h as the characteristic size of the UC and l as the characteristic wavelength of the deformation of the macroscopic material.

In this paper, we will first use this modeling technique to construct a new micromechanics model for effective thermoelastic properties including elastic properties, coefficients of thermal expansion (CTEs), and specific heat for heterogeneous materials. Then we will implement the theory using the finite element method in the computer program VAMUCH to provide engineers with a general-purpose micromechanics tool for thermoelastic micromechanical analysis of composite materials.

2. Theoretical formulation

We need to use three coordinate systems: two Cartesian coordinate systems $\mathbf{x} = (x_1, x_2, x_3)$ and $\mathbf{y} = (y_1, y_2, y_3)$, and an integer-valued coordinate system $\mathbf{n} = (n_1, n_2, n_3)$; see Fig. 2. We use x_i as the global coor-