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Finite viscoplasticity of amorphous glassy polymers in the logarithmic strain space

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ABSTRACT

The paper outlines a new constitutive model and experimental results of rate-dependent finite elastic–plastic behavior of amorphous glassy polymers. In contrast to existing kinematical approaches to finite viscoplasticity of glassy polymers, the formulation proposed is constructed in the logarithmic strain space and related to a six-dimensional plastic metric. Therefore, it a priori avoids difficulties concerning with the uniqueness of a plastic rotation. The constitutive framework consists of three major steps: (i) A *geometric pre-processing* defines a total and a plastic logarithmic strain measures determined from the current and plastic metrics, respectively. (ii) The *constitutive model* describes the stresses and the consistent moduli work-conjugate to the logarithmic strain measures in an analogous structure to the *geometrically linear theory*. (iii) A *geometric post-processing* maps the stresses and the algorithmic tangent moduli computed in the logarithmic strain space to their nominal, material or spatial counterparts in the finite deformation space. The analogy between the formulation of finite plasticity in the logarithmic strain space and the geometrically linear theory of plasticity makes this framework very attractive, in particular regarding the algorithmic implementation. The flow rule for viscoplastic strains in the logarithmic strain space is adopted from the celebrated double-kink theory. The post-yield kinematic hardening is modeled by different network models. Here, we compare the response of the eight chain model with the newly proposed non-affine micro-sphere model. Apart from the constitutive model, experimental results obtained from both the homogeneous compression and inhomogeneous tension tests on polycarbonate are presented. Besides the load–displacement data acquired from inhomogeneous experiments, quantitative three-dimensional optical measurements of the surface strain fields are carried out. With regard to these experimental data, the excellent predictive quality of the theory proposed is demonstrated by means of representative numerical simulations.

1. Introduction

Amorphous glassy polymers have been widely employed in various practical application areas that cover automotive and construction industry, electronics, optical devices and medical technology, to mention a few. The broad spectrum of application is due to their good processing features, high energy absorption capacity under impact loadings, lower weight relative to glass and excellent optical properties. The geometry of the products used in the above mentioned practical applications is generally three-dimensional and has varying aspect ratio and dimensional scales. Apart from the geometrical challenge, amorphous glassy polymers exhibit rate-dependent finite elastic–plastic material behavior. The elasto-viscoplastic response stems from the inherent disordered micro-structure of the material that is formed by linear polymer chains existing in the

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"frozen-in" state. In contrast to elastomers or thermosets, they are generally not cross-linked by chemical bonds but their network structure is rather formed by physical junctions, the so-called *entanglements*. This intrinsic micro-structure brings along the rate and temperature effects prevailing in the material behavior. The finite elasto-viscoplastic behavior is not specific only to tough polymers but is also observed in brittle polymers on a much smaller scale, especially in the course of crazing. For this reason, both a sound three-dimensional constitutive model accounting for the complicated material behavior and the associated effective numerical algorithm for finite element simulations are of great importance.

1.1. Mechanical behavior of amorphous glassy polymers

Probably, the most illustrative example for the finite elasto-viscoplastic behavior of ductile glassy polymers is the so-called *cold drawing* process. Fig. 1 depicts the tensile load–displacement curve of a dumbbell-shaped polycarbonate (Makrolon 2607) test piece (ISO 527-Type 1B) undergoing cold drawing. The experiment was conducted on a MTS servohydraulic uniaxial testing machine at a constant cross-head speed $\dot{u} = 2$ mm/min and room temperature. The representative stages of the deformation are labeled from (a) to (h) on both the load–displacement diagram and the snapshots of the deformed specimen in Fig. 1. Combination of the load–displacement diagram and the associated images shows the initation, stabilization and propagation of the neck. Apart from these images, the quantitative principal stretch contours corresponding to the stages (a)–(f) of the experiment are presented in Fig. 2. The contours were obtained by post-processing the recognized part of each associated image whose periphery is highlighted with the solid line in Fig. 2. The commercial software ARAMIS, which is based on the optical measuring technique, the so-called *grating method*, was used to capture the three-dimensional strain field evolving on the surface of the dumbbell-shaped specimens during deformation. Further details concerning the preparation of specimens and the procedure followed for inhomogeneous strain field measurements are outlined in Section 4.

In Fig. 1, the initial linearly elastic part of the load–displacement curve bounded by the level *a*) falls into the range of small deformations. In the succeeding interval, between the stages (a) and (b), the curve gradually becomes non-linear and exhibits viscoelastic charateristics as shown in Fig. 1. At these stages of the extension, the strain field along the specimen is measured to be uniform, see Fig. 2a and b. The highest load level (b) attained at the end of the non-linear viscoelastic part is generally called the *macro-yield point*. Any unloading below this point does not result in significant hystereses or permanent strains, see Lu and Ravi-Chandar (1999). It has been shown that the pre-yield viscoelastic behavior is essential to elucidate the non-linear unloading and creep response of the material. For more detailed discussions concerning the phenomenon, the reader is referred to the recent works of Hasan and Boyce (1995) and Anand and Ames (2006). Further extension beyond the yield point (b) leads to inelastic strain localization regions that generally appear in the form of micro shear bands causing a softening in the load–displacement response, see Tomita et al. (1997) and Lu and Ravi-Chandar (1999). These bands then

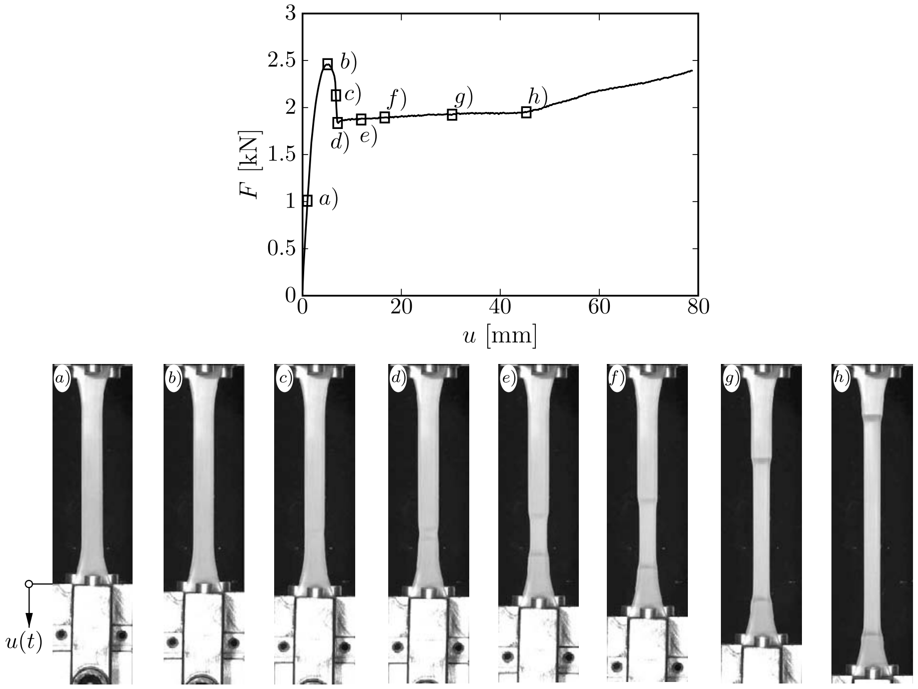

Fig. 1. Load–displacement diagram and the snapshots of a dumbbell-shaped polycarbonate (PC) specimen subjected to extension at a constant cross-head speed $\dot{u} = 2$ mm/min and room temperature. The selected stages of deformation labeled from (a) to (h) depict the process of initiation, stabilization, and propagation (cold drawing) of the neck.