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A nonequilibrium irreversible thermodynamics model for material damping

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Abstract

Material damping in a continuum, which inherently involves multiple coupled irreversible thermodynamic processes, is associated with irreversible interchanges of various forms of energy. In this paper, a hybrid framework of internal state variables and extended state variables is proposed to formulate a material damping model for complex materials. For a typical simple material, damping model is developed in the framework of extended irreversible thermodynamics. Nonequilibrium quantities, i.e. thermodynamic fluxes, are introduced as extended state variables to supplement for the description of a local nonequilibrium state. The relaxation characteristics of these thermodynamic fluxes, which are a symbol of nonequilibrium characteristics, are modeled by first-order relaxation equations. For a typical system, we introduced heat flux and nonequilibrium (viscous) stresses as thermodynamic fluxes corresponding to irreversible heat transport and viscous processes. Coupling between equilibrium and nonequilibrium mechanical and thermal fields are modeled. Explicit expressions for specific free energy, specific entropy, and specific internal energy are derived. The energy balance equation that governs interchanges of various forms of energy is then obtained to complete the formulation. We have investigated various limiting cases of the developed model and related these limiting cases to typical thermodynamic damping models. We have applied the model to study the dissipation characteristics of longitudinal vibrations of a rod and then compared with other damping models.

Keywords: Damping; Thermo-mechanical response; Dissipation; Classical irreversible thermodynamics; Rational thermodynamics; Extended irreversible thermodynamics

1. Introduction

Even though, we understand the elastic behavior of a structure, reasonably accurately, we do not understand the damping in the structure to the same level. The damping is a very desirable structural characteristic to attenuate vibrations, increase the fatigue life of a structure, and eliminate the undesired noise. The damping of a structure is primarily due to the material damping and possible dissipation effects by joints, fasteners and

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interfaces. To date, it is very difficult to accurately estimate the damping in a structure and therefore estimate the associated small changes in natural frequencies and other dynamic characteristics of the structure. However, these small changes are very important in many applications such as health monitoring of structural systems and estimating the residual strength of a structure. A precise material damping model of highly physical insight will lead to structural dynamic models of improved accuracy for simulation, system diagnostics and identifications, and design of active/passive control systems.

1.1. Mechanisms of material damping

Let us consider an elastic cantilever beam that is isolated in vacuum. When it is released from an initially deflected position, the beam vibrates with gradually decreasing amplitudes and finally comes to rest at an unstrained position. This simple case manifests the characteristics of material damping. Here, mechanical energy is converted into another form of energy, i.e., the heat, which is stored in the closed system, through a thermodynamic process, i.e., the thermoelastic process. In the thermoelastic process, it involves a coupling between the equilibrium thermal and mechanical fields. For a material of viscous characteristics, rather than the thermoelastic damping, viscous process and its coupling with other irreversible processes attribute to dissipation and damping. For a more general material, multiple irreversible processes at disparate scales may also contribute to the total dissipation. Thus, a precise damping model needs to describe various irreversible processes that can be at disparate scales, their inter-field coupling, and conversion of various forms of energy. This has to be developed in the framework of irreversible thermodynamics.

1.2. Framework of irreversible thermodynamics

Before we start formulating our damping model, we give a brief discussion on how we justify our thermodynamics framework from various irreversible thermodynamics theories. Starting from 1930s, Onsager, Prigogine, Eckart, Mexiner, de Groot, Mazur and others succeeded in formulating the principles of irreversible thermodynamics by making some restrictive assumptions (Onsager, 1931a,b; Onsager and Machlup, 1953). The concept of "state" is extended from a global description of a continuum in thermostatics (Buchdahl, 1966) to a local description of material point in the continuum. In other words, every material point that constructs the continuum is assumed approximately close to a local thermodynamic equilibrium state at any given instant (Fung, 1965; Prigogine, 1955; Jou et al., 1996). Therefore, state variables like temperature that are well defined in thermostatics for a global system can be used to specify a local equilibrium state at a material point. Today, this theory is known by the name *classical irreversible thermodynamics* (CIT). Besides the classical set of state variables, Onsager et al. introduced thermodynamic fluxes to describe irreversible processes. At any instant, thermodynamic fluxes instantaneously respond to the generalized forces by a linear relation that holds the Onsager–Casimir reciprocal principle (Onsager, 1931a,b; Casimir, 1945). These thermodynamic fluxes are not considered as additional state variables; therefore, do not appear in state functions like internal energy, free energy, etc., as defined in thermostatics. In 1943, Bridgeman addressed a balance law of entropy (i.e., the evolution of entropy): "(net entropy leaving a close region) = (entropy created within the region) − (increase of entropy localized in this region)". In other word, the rate of change of entropy within a region is contributed by entropy flux through the region and entropy production created inside the region. In CIT, entropy flux only depends on heat flux, which is the thermodynamic flux for irreversible heat transfer process. The rate of entropy production is in a bilinear form of fluxes and their associated generalized forces. The nonnegativity of rate of entropy production, i.e. the dissipation inequality or Clausius–Duhem inequality, grants the irreversibility of dissipative processes and states the second law of CIT.

Rational thermodynamics (RT) was initially introduced by Truesdell, Coleman, Noll and others to enlarge the scope of the application of nonequilibrium thermodynamics beyond CIT (Truesdell, 1984; Coleman and Gurtin, 1967; Day, 1972; Jou et al., 1996). In RT, materials are assumed having memory. This means, at any given instant, dependent variables cannot be determined by only instantaneous values of independent variables, but by their entire histories. In some ways, this modifies the concept of state in CIT, where a state is well defined and determined by a set of measurable variables at only current instant. In RT, complementary variables (like internal energy, heat flux, stress tensor and entropy) are related to the entire history of indepen-