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Finite deformation higher-order shell models and rigid-body motions

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Abstract

This paper focuses on the developments of higher-order shell models by employing the new concept of *interpolation surfaces* (*I*-surfaces) inside the shell body. We introduce N ($N \geqslant 3$) *I*-surfaces and choose the values of displacements with correspondence to these surfaces as fundamental shell unknowns. Such choice allows, first, to develop various higher-order shell models in a very compact form and, second, to derive non-linear strain–displacement relationships, which are completely free for arbitrarily large rigid-body motions. The general $3N$-parameter shell model is proposed in the framework of the Lagrangian description. The special 9, 12 and 15 parameters cases (corresponding to $N = 3$, 4 and 5) have been dealt in detail. The proposed shell models account for thickness stretching and the complete 3D constitutive equations are utilized. The displacement vectors of equally located *I*-surfaces for each model are represented in a *convected* curvilinear coordinate system allowing one to develop directly finite deformation geometrically exact shell finite elements which could be discussed in future works.

Keywords: Finite deformation; Higher-order shell theory; Rigid-body motion; Interpolation surfaces; Lagrange polynomials

1. Introduction

One of the main requirements of any shell theory that is intended for application to computational mechanics (e.g. a finite element method) is that it must lead to strain-free modes for arbitrary rigid-body motions. The adequate representation of rigid-body motions is a necessary condition if good both accuracy and convergence properties are required. Therefore, when an inconsistent shell theory is utilized to develop any finite element, erroneous straining modes under rigid-body motions can be appeared. This problem has been studied for the classical Kirchhoff–Love shell theory by Cantin (1968) and Dawe (1972). Further developments for the finite deformation 6-parameter homogeneous and layer-wise shell theories based on the Mindlin kinematics have been done by Kulikov (2004) and Kulikov and Plotnikova (2003, 2006a). This work proposes higher-order

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shell theories on the basis of the strain–displacement equations, which exactly represent arbitrarily large rigid-body motions in a *convected* curvilinear coordinate system.

A large number of works has been already done to develop the finite deformation higher-order shell formulation (Librescu, 1987; Parisch, 1995; Sansour, 1995; Basar et al., 2000; El-Abbasi and Meguid, 2000; Sansour and Kollmann, 2000; Brank et al., 2002; Krätzig and Jun, 2003; Brank, 2005; Arciniega and Reddy, 2007) with thickness stretching. These articles except for purely theoretical contributions of Librescu (1987) and Krätzig and Jun (2003) are devoted to the 7-parameter shell theory in which the transverse normal strain varies at least linearly through the shell thickness. This fact is of great importance since the 6-parameter shell formulation based on the complete 3D constitutive equations exhibits thickness locking as mentioned in works (Kim and Lee, 1988; Buchter et al., 1994; Park et al., 1995; Kulikov, 2001; Sze, 2002; Kulikov and Plotnikova, 2006a; Carrera and Brischetto, 2007). The errors caused by thickness locking do not decrease with the mesh refinement because the reason of stiffening lies in the shell theory itself rather than the finite element discretization. We refer to review papers of Carrera (2002, 2003), where one may read that a conventional way for developing the higher-order shell formulation is to utilize either quadratic or cubic series expansions in the thickness coordinate and to choose as unknowns the generalized displacements of the reference (middle) surface. A Unified Formulation is also discussed in the latter paper (Carrera, 2003), which permits to develop plate/shell theories in terms of a few fundamental nuclei whose forms do not depend on the order of the used expansion.

In this paper, the higher-order shell models are developed by using the new concept that employs N *interpolation surfaces* (I-surfaces) inside the shell body, in order to choose displacements of these surfaces as fundamental unknowns. Such choice of displacements with the consequent use of the Lagrange polynomials in the thickness direction allows one to represent all three higher-order shell formulations developed in a compact form (similar to that used by Carrera, 2003) and to derive non-linear strain–displacement equations which are completely free for large rigid-body motions. Taking into account that displacement vectors of I-surfaces are introduced and resolved in the reference surface frame the proposed higher-order shell formulations are very promising for developing high performance finite rotation geometrically exact shell elements (Kulikov and Plotnikova, 2006b, 2007). The term "geometrically exact" reflects the fact that coefficients of the first and second fundamental forms, and Christoffel symbols are taken exactly at every Gauss integration point. Therefore, no approximation of the reference surface is needed. The advantage of the geometrically exact shell element formulation for coarse meshes is discussed in aforecited papers.

It should be mentioned that in some works (see e.g. Parisch, 1995; Kulikov, 2001, 2004; Carrera, 2003; Kulikov and Plotnikova, 2003, 2007) on solid-shell elements the displacement vectors of the bottom and top surfaces are also utilized. An idea of this approach can be traced back to the contribution of Schoop (1986). But in our higher-order shell formulation the use of bottom and top surfaces has a principally another mechanical sense: these are just a part of a set of I-surfaces inside the shell body. To substantiate this point of view, we remark that respectively $N = 3$, 4 and 5 equally located surfaces have been chosen as I-surfaces (such assumption is not mandatory, the I-surfaces could be also located with different relative distances). Thus, the resulting shell models are characterized by $3N$ parameters, i.e., we deal with 9, 12 and 15-parameter shell models.

It is of interest to notice that there is a connection between the proposed higher-order solid-shell model and the Lagrange continuum-based element (Zienkievicz and Taylor, 2000). The main difference consists in the fact that in our solid-shell formulation the Lagrange polynomials in the thickness direction are utilized before starting the discretization procedure by the finite element method. Thus, we deal with the stress resultant solid-shell theory, which permits one to derive equilibrium equations in both weak and strong forms. The latter can be useful for the analytical developments. Besides, the proposed solid-shell formulation based on the I-surface technique clears the way for developing efficient geometrically exact higher-order shell elements.

The attention of the present work has been restricted to the full description of fundamentals of the non-linear shell theory based on the I-surfaces including a weak form of equilibrium equations. The finite element formulations and their applications to homogeneous structures as well as the extension to laminated composite structures could be conveniently discussed in future developments.