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Bounds for shakedown of cohesive-frictional materials under moving surface loads

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Abstract

In this paper, shakedown of a cohesive-frictional half space subjected to moving surface loads is investigated using Melan's static shakedown theorem. The material in the half space is modelled as a Mohr–Coulomb medium. The sliding and rolling contact between a roller and the half space is assumed to be plane strain and can be approximated by a trapezoidal as well as a Hertzian load distribution. A closed form solution to the elastic stress field for the trapezoidal contact is derived, and is then used for the shakedown analysis. It is demonstrated that, by relaxing either the equilibrium or the yield constraints (or both) on the residual stress field, the shakedown analysis leads to various bounds for the elastic shakedown limit. The differences among the various shakedown load factors are quantitatively compared, and the influence of both Hertzian and trapezoidal contacts for the half space under moving surface loads is studied. The various bounds and shakedown limits obtained in the paper serve as useful benchmarks for future numerical shakedown analysis, and also provide a valuable reference for the safe design of pavements.

Keywords: Shakedown; Cohesive-frictional materials; Cyclic load; Hertzian contact; Trapezoidal contact; Analytical solution; Pavement design

1. Introduction

The static shakedown theorem proposed by Melan (1938), together with the kinematic shakedown theorem proposed by Koiter (1960), constitute the cornerstone of shakedown analysis for elastoplastic structures under cyclic loading. While originally used to address material behaviour with simple assumptions such as geometric linearity, elastic perfectly plastic constitutive relations and associated flow, the shakedown theorems have been extended to cover a broad category of applications that account for the effects of high temperature, strain or work hardening, nonlinear geometry, dynamic behaviour, and non-associated plastic flow. Numerical methods such as the finite element method have also been employed to predict shakedown behaviour in various materials for general cases. Shakedown theory has thus become a useful tool in the design of a wide range

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of practical applications such as railway foundations, pavement engineering, roller bearings, and the machining industry. Comprehensive reviews of the development of shakedown theory can be found in various monographs such as those of König and Maier (1981), Polizzotto (1982), Johnson (1985) and König (1987), as well as more recent research papers (see, for example, Ponter et al., 1985, 2006; Pycko and Maier, 1995; Maier, 2001; Polizzotto et al., 2001; Zouain and Silveira, 2001; Bousshine et al., 2003; Feng and Sun, 2007; Polizzotto, 2007; Pham, 2007 and references therein).

In pavement engineering, the evaluation of road performance requires proper assessment of the permanent deformation and fatigue under moving traffic loads. The complex nature of the problem clearly requires accurate theoretical models in order to simulate the actual failure mechanics observed in pavement engineering (Brown, 1996). In practice, it is economically desirable to construct pavements that can sustain stress levels well beyond the elastic limit of their constituent materials. In particular, it is of major importance to determine whether a given pavement structure, when subjected to a large number of load cycles, will experience progressive accumulation of plastic strains and gradual failure, or whether the increase in plastic strains will cease to occur, thereby leading to a stable response or shakedown. Field observations indicate that many pavements do in fact shakedown rather than deform continuously. The use of the shakedown theorems can thus enable the long term behaviour of a pavement to be determined without resorting to computationally expensive step-by-step analyses. As a consequence, shakedown theory has received much attention from researchers in the field of pavement engineering over the past two decades – see, for example, the work of Sharp and Booker (1984), Collins and Cliffe (1987), Collins et al. (1993), Yu and Hossain (1998), Collins and Boulbibane (2000), Yu (2005) and Krabbenhøft et al. (2007a).

For a given structure and load regime, the shakedown load factor depends on the elastic parameters and the yield limits of the materials, not on the actual way the system evolves before the shakedown condition is reached (Martin, 1975; Stein et al., 1992). In order for shakedown theory to provide useful pavement design information, the Mohr–Coulomb yield criterion has long been preferable to the Tresca criterion for the description of the cohesive-frictional nature of pavement materials, but this gives rise to a number of complications in analytical and numerical computations. Sharp and Booker (1984) proposed an elegant method of conics to handle Mohr–Coulomb materials in shakedown analysis. This was studied further by Collins and Cliffe (1987) in conjunction with kinematic upper bounds for the shakedown limit. The finite element method, in tandem with linear and nonlinear programming techniques, has also been used to compute shakedown limits numerically (see, e.g., Shiau, 2001; Boulbibane and Ponter, 2006; Li and Yu, 2006). When applying Melan's theorem to cohesive-frictional materials, however, considerable confusion exists that may give rise to inaccurate and inconsistent predictions of the shakedown limit. Specifically, some of the constraints on the residual stresses that are necessary in deriving rigorous shakedown limits are often inadvertently neglected. As pointed out by Krabbenhøft et al. (2007a), and demonstrated again here, this always leads to upper bounds to the static shakedown limit.[1]

The *first* constraint that is frequently neglected in the literature is *the yield condition* on the residual stress field. According to Melan's theorem, this constraint is necessary in deriving the shakedown limit for cyclic loading of a pavement. For such a pavement, the moving loads on the road surface constitute a typical two-point load domain with zero being one of the load vertices. Once the external load becomes zero, the yield constraint on the combined (total) stresses in Melan's theory will degenerate to a yield constraint on the residual stresses alone. Thus, to ensure the whole stress history lies within the yield surface, the yield constraint must be imposed on the residual stresses. One may argue that by checking the yield condition on the total stresses this scenario should be covered automatically as a special case. Depending on the procedure used to compute the shakedown limit, this is not always sufficient (as discussed in detail in Section 4). Research that recognises this key point in pavement shakedown analysis includes the work of Sharp and Booker (1984), Collins and Cliffe (1987) and Krabbenhøft et al. (2007a). Collins and Cliffe (1987) appear to be the first to explicitly indicate that the residual stresses must satisfy the yield condition when computing the shakedown limit. They remark that the *positive* shakedown load factor must be obtained with *a residual stress that lies between the uniaxial compression and tension limits*. This requirement is precisely the yield constraint on the

[1] Provided an exact elastic solution is employed and the discretisation error is small.