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Large strain elasto-plastic model of paper and corrugated board

Anders Harrysson, Matti Ristinmaa *

Division of Solid Mechanics, Lund University, Box 118, S-221 00 Lund, Sweden

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Abstract

An anisotropic elasto-plastic constitutive model of paper material is presented. It is formulated in a spatial setting in which anisotropic properties are accounted for by use of structural variables. A multiplicative split of the deformation gradient is employed to introduce plasticity. A similar approach is used to model the plastic deformation of the substructure. The yield surface adopted is based on the Tsai–Wu failure criterion, used previously to model failure of paper material. A non-associated plasticity theory is employed to calibrate the model to experimental data. It turns out that a multi-axial loading situation is needed to calibrate the model and here a biaxial tension test is audited. The model was implemented into a finite element environment and the creasing process of a corrugated board panel is investigated.

Keywords: Plasticity; Anisotropy; Paper material; Thermodynamics; Finite strains

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1. Introduction

Packaging paper is a generic name for paper material of all types used for packaging goods. The material known as corrugated board is produced by a converting process in which two or more layers are laminated, cf. Fig. 1. The flat top and bottom layers are called liners and the corrugated core is referred to as fluting. Corrugated board is frequently used for making boxes for the transport of goods and the like, is one of the most used packaging material. Its low cost per unit weight, the possibility of recycling and the high stiffness per unit weight makes it an attractive material. During the lifetime of a package, the material of which it is constructed will be exposed to mechanical loading during for instance transportation and storage. In the past, attempts have been made to predict the load-carrying capacity of corrugated box, cf. the pioneer work by McKee et al. (1963). More recent work addressing this problem can be found in Patel et al. (1997), Nyman (2000) and Nordstrand et al. (2003). Still more recently in the work of Biancolini (2005) and Isaksson and Hägglund (2005) the finite element method was used to gain better understanding of how a corrugated board panel deforms during sever mechanical loading. It has been found that even during the box manufacturing process itself the corrugated board may be severely deformed, particularly in the folding areas. The modeling of this process will be considered here and therefore a material model for paper will be the main concern in this paper.

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* Corresponding author. Fax: +46 46 2223115.
E-mail address: Matti.Ristinmaa@solid.lth.se (M. Ristinmaa).

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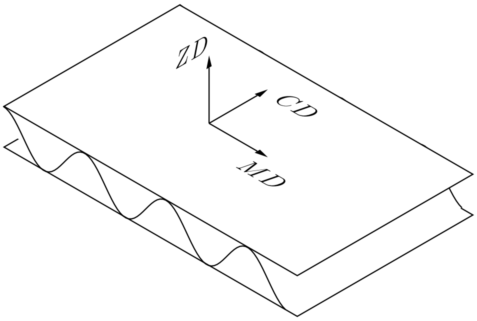

Fig. 1. Single wall corrugated board panel. The material directions of the paper are indicated.

The main building blocks of packaging paper are cellulose fibers, consisting primary of wood fibers, although other materials are sometimes employed as well. The manufacturing process usually involves the dewatering of a cellulose fiber suspension on a web. The fibers have the inherent capability of bonds being formed between them without the use of any additives. Due to the manufacturing of the separate paper layers, the fibers tend to become aligned to the direction in which the web is running. This direction is usually referred to as the machine direction (MD). The direction perpendicular to this direction in the plane of the web is called the cross direction (CD). The third direction is the out-of-plane direction (ZD), cf. Fig. 2. The strength of the bonds and the longitudinal properties of the fibers are the main factors for the in-plane mechanical properties of the paper sheet. The mechanical properties in the out-of-plane direction are related to the fiber properties perpendicular to the longitudinal direction and the bond strength. Due to the orientation of the fibers in the paper material, the mechanical response will differ depending upon the loading direction. Since the fibers tend to become oriented in the MD direction this direction also is the 'strongest' direction, i.e. higher Young's modulus, yield stress etc. The material properties in CD are about two to four times lower than in MD. Since the in-plane and out-of-plane mechanical properties are governed by rather different physical mechanisms, the mechanical out-of-plane properties are very different from the in-plane properties. Stiffness and yield stress are of the order of two lower than the in-plane properties.

As known from material testing, cf. Steenberg (1949), paper show a highly non-linear response even when exposed to only moderate deformations. One can note as well that unloading from the non-linear region introduces non-recoverable strains. Such observations motivate the use of plasticity theory. Further and very important factor when modeling paper material are the directional dependent properties. Attempts to model the mechanical properties of paper material have been made recently by Castroa and Ostoja-Starzewski (2003) as well as Xia et al. (2002) and Mäkelä and Östlund (2003) who considered elasto-plastic properties of paper and Isaksson et al. (2004) how also considered damage.

The present study takes up a large strain elasto-plastic material model of paper based on an orthotropic hyperelastic model, orthotropic yield surface and hardening model. To account for the different yield stresses in tension and compression, an approach similar to that of Shih and Lee (1978) has been adopted. However, since paper do

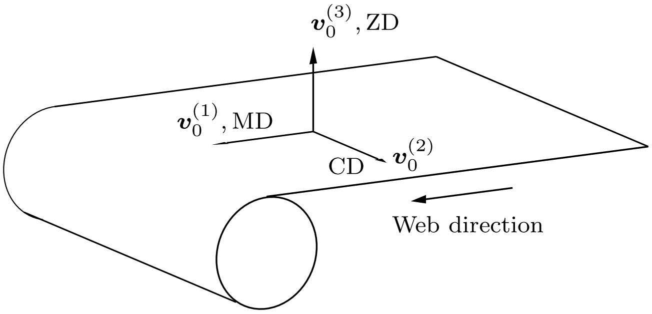

Fig. 2. Illustrations of the different material directions of a single material layer due to manufacturing process. Here $v_0^{(1)}$, $v_0^{(2)}$ and $v_0^{(3)}$ denotes the base vectors defining the directions corresponding to MD, CD and ZD, respectively.