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R-curve and size effect in quasibrittle fractures: Case of notched structures

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Abstract

Within the framework of Bažant's theory, the size effect on nominal strength of notched structures deduced from a size-dependent R-curve is proposed. It is shown that the expected size effect is more complicated than the one proposed in Bažant's Size Effect Law (SEL) and especially in the crossover regime. As a function of the fracture parameters describing the R-curve, two kinds of size effect on the resistance at peak load are possible and lead to three different scalings on the nominal strength. We argue that these expected size effects are mainly driven by the value of the scaling exponent characterizing the size effect on the critical crack length increment and on the critical resistance assumed in the R-curve behavior. The three resulting size effects on the nominal strength are compared to Bažant's SEL. It appears that, if Bažant's SEL always underestimates nominal strength and consequently provides a safety design of structures, an optimal design should take into account the size effect on the R-curve and their consequences on the size effect on the nominal strength especially for large structures sizes.

Keywords: Quasibrittle fracture; R-curve; Scaling exponents; Crack length at peak load; Size effect

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1. Introduction

In solid mechanics, an essential scaling problem is the effect of the structure size on its nominal strength. This effect is particularly important in the case of quasibrittle materials which are characterized by the existence of a large fracture process zone (FPZ) where various toughening mechanisms take place such as microcracking, crack branching or crack bridging. Materials as different as concretes, mortar and rocks, some composites and wood belong to this category. In notched structures, the fracture behavior of quasibrittle materials is usually characterized by a more or less pronounced rising resistance curve, commonly called R-curve (Lawn, 1993). This R-curve behavior emphasizes stress redistributions and stored energy release which take place in such large FPZ producing large stable crack growth before failure.

Since 1984, Bažant and co-workers (Bažant, 1984; Bažant, 1997a,b; Bažant, 2000) have shown that in the case of quasibrittle materials, contrary to what happens for Weibull's statistics (Weibull, 1939), the size effect

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is linked to the very existence of the R-curve behavior. Within the framework of Bažant's theory (Bažant, 1997a,b), the size effect can be described in the case of geometrically similar notched structures (with geometrically similar initial cracks) of different characteristic sizes D (dimension) by introducing a nominal stress:

$$\sigma_{\mathrm{N}} = c_{\mathrm{N}} \frac{P}{D^2}, \tag{1}$$

where P is the external load applied to the structure (load independent of the displacement) and c_{N} is a coefficient introduced for convenience. When $P = P_{\mathrm{u}}$, which corresponds to the ultimate load or peak load, σ_{N} is called the nominal strength of the structure. From an energy-based asymptotic analysis founded on a single R-curve, i.e., independent of the specimen size, (Bažant (1997b)) has shown that, in a first order asymptotic approximation, the nominal strength σ_{N} can be estimated as a function the characteristic size D as:

$$\sigma_{\mathrm{N}} = \frac{Bf_t}{\sqrt{1 + \frac{D}{D_0}}}, \tag{2}$$

where (Bf_t) has the dimension of a stress (Pa) and D_0 is the crossover size (m) between both asymptotic behaviors. In the case of small structure sizes, i.e., $D \ll D_0$, $\sigma_{\mathrm{N}} \simeq Bf_t = \text{const}$: no size effect is expected. Indeed, for these small structure sizes, the fracture process zone is expected to occupy the whole volume of the structure, inducing no stress concentration. As a consequence, failure occurs with no crack propagation: this is the domain of strength theory. For large structures sizes, i.e., $D \gg D_0$, contrary to what happens for small sizes, $\sigma_{\mathrm{N}} \sim D^{-1/2}$ which is the size effect expected from linear elastic fracture mechanics (LEFM). A possible justification is that in large structures, the process zone is expected to lie within only an infinitesimal volume fraction of the body and hence, the stress and displacements fields surrounding the FPZ are the asymptotic elastic fields considered in LEFM.

However, despite the success of Bažant's Size Effect Law (SEL) (Eq. (2)) to describe size effect of quasibrittle materials, the crossover regime between both asymptotic behaviors (estimated from the intermediate asymptotic theory) does not appear accurately defined. This point deserves some more thinking, especially since this is usually the range of the experimental values from which the SEL is entirely defined. On the other hand, it has been shown recently that the R-curve might be size-dependent (Morel et al., 2002a,b) contrary to what is assumed in Bažant's SEL where any size effect on R-curve is considered.

In this study, within the framework of Bažant's theory (Bažant, 1997a,b), the size effect on nominal strength is studied in the case of geometrically similar notched structures (with similar initial cracks) characterized by one dimension D. We show that the size effect on nominal strength deduced from an analytical size-dependent R-curve appears more complicated than the one proposed in Bažant's SEL and especially in the crossover regime. In Section 2, the more appropriate mathematical expression of the R-curve with respect to size effect is studied and discussed. An R-curve expression describing the size effects on the critical crack length increment and on the critical resistance is proposed. In Section 3, the implications on the size effect on the resistance at peak load are discussed in relation to the values of the scaling exponent describing the R-curve. The size effect on the nominal strength are then investigated in Section 4 as a function of the different scaling obtained for the resistance at peak load. Finally, the results are discussed in Section 5 and a comparison to the prediction of the Bažant's SEL is performed.

2. R-curve and effective length of the FPZ

In notched structures, the fracture of quasibrittle materials can be successfully described within the framework of an equivalent linear elastic approach. Within this framework also called 'equivalent LEFM' the increase of the structure compliance due to the FPZ development is attributed to the propagation of an elastically equivalent crack (Bažant and Kazemi, 1990; Bažant, 1997a; Morel et al., 2005) which gives (according to LEFM) the same structure compliance as the actual crack with its fracture process zone. Thus, energy stored in the structure can be characterized by the complementary energy W^*:

$$W^* = \frac{P^2}{E'b} f\left(\frac{a}{D}\right), \tag{3}$$