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Electromechanical behaviour of a finite piezoelectric layer under a flat punch

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ABSTRACT

This paper solves the problem of a smooth and frictionless punch on a piezoelectric ceramic layer. Different electrical boundary conditions that employ conducting or insulating punches are presented. The stress and electric displacement intensity factors are used to characterize the electromechanical fields at the punch tip. The field intensity factors are obtained numerically for finite layer thickness. Effects of the thickness of the piezoelectric layer on the stress and electric displacement, and the stress and electric displacement intensity factors at the punch tip are discussed. Solution technique for two identical and collinear surface punches on the piezoelectric layer is also provided and the effect of relative distance between the punches is investigated. Numerical results for some interesting special cases, such as conducting punch and insulating punch, and infinite piezoelectric layer thickness, are presented.

1. Introduction

Electroelastic interaction of piezoelectric ceramics is important in smart materials and devices. Because of the intrinsic brittleness of piezoelectric ceramics, stress concentrations near electro-mechanical contact regions, caused by the inharmonious contacts between piezoelectric ceramics and other components, could result in failure of piezoelectric components.

There have been some previous attempts at the development of continuum solutions for punch contact and electrode problems with piezoelectric ceramics (Melkumyan, 2005, 2007; Hausler and Balke, 2005; Mo et al., 2007; Narita et al., 2004, 2007; Shindo et al., 2004). In particular, Sosa and Castro (1994) obtained solutions for point force and point charge acting on an orthotropic piezoelectric half-plane through a state space methodology in conjunction with Fourier transforms. A concentrated lateral shear force acting on a transversely isotropic piezoelectric half-space was studied by Wang and Zheng (1995). The solution of point forces and point charges acting on the boundary of a piezoelectric half-space has been obtained by Ding et al. (1996). The two-dimensional contact problem of a piezoelectric half-plane was studied and solutions for loads acting on the boundary of an anisotropic piezoelectric half-plane were also provided (Fan et al., 1996). The solution of the problem of a rigid punch with a parabolic cross-section and flat base that is forced into an elastic piezoelectric ceramic half-space has been derived in explicit form (Podilchuk and Tkachenko, 1999). Ding et al. (2000) investigated a series of three-dimensional contact problems including a spherical contact, a conical indenter and an upright circular flat punch on a transversely isotropic piezoelectric half-space. A local–global stiffness matrix methodology for the response of a composite half-plane, arbitrarily layered with isotropic, orthotropic or monoclinic plies, to indentation by a rigid parabolic punch was extended to accommodate the presence of honeycomb or piezoelectric layers (Zhang et al., 1999). A three-dimensional study of a rigid smooth punch bonded to a transversely isotropic piezoelectric half-space has been conducted (Chen, 2000). In addition, a rigid conical punch indenting a transversely isotropic piezoelectric half-space was considered (Chen et al.,

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1999). A general theory was presented for axisymmetric indentation of piezoelectric solids within the context of fully coupled, transversely isotropic elasticity models (Giannakopoulos and Suresh, 1999). An exact solution for a flat smooth punch applied on a piezoelectric half-plane was derived and the distribution of pressure under the punch has been found (Hao, 2003). The fracture behavior of a piezoelectric ceramic under applied electric fields through experimental and theoretical characterizations by modified small punch (MSP) tests (Shindo et al., 2003). Further, finite element was used to compare with experiments using the modified small punch (MSP) technique (Magara et al., 2004). Recently, a circular surface indenter on a piezoelectric layer (Wang and Han, 2006), a rigid punch indenting an anisotropic piezoelectric half-space (Li and Wang, 2006), a frictionless contact on arbitrarily multilayered piezoelectric half-planes (Ramirez, 2003), and a rigid frictionless parabolic punch (Ramirez, 2006) were investigated. The micro-scale adhesive contact behavior of a spherical rigid punch on a piezoelectric half-space was investigated and the effect of piezoelectric property on the adhesive contact behavior of piezoelectric materials was explored (Chen and Yu, 2005). An exact solution for the Green's functions of a half-infinite piezoelectric solid based on the exact electric boundary conditions at the interface between the solid and the air medium was developed (Gao and Noda, 2004). Three-dimensional transversely isotropic piezoelectric media under punch loading was investigated by boundary integral equation method (Zhao et al., 2005). All of these papers considered the rigid contact between the punch and the piezoelectric materials. Electric and elastic behaviors of a piezoelectric ceramic was investigated for a soft, charged surface electrode (Li and Lee, 2004).

The above-mentioned studies on contact and indentation of piezoelectric materials were mainly limited to elliptical and circular contact shapes. Moreover, previous studies on electric and elastic behaviors are mainly for infinite thickness of the piezoelectric layer. However, comparing with the indenter or punch size, a practical piezoelectric medium may have finite thickness, rather than infinite thickness. No straight punch or indenter on a piezoelectric layer of finite thickness has been considered so far. It is therefore necessary to establish a general model for the finite layer thickness from which results for the infinite layer can also be derived.

Motivated by these considerations, this paper investigates the problem of a piezoelectric layer with a straight punch on its surface. The analysis of the problem leads to a system of singular integral equations, which can be solved numerically to obtain the electromechanical fields in the piezoelectric layer. Closed-form solution is obtained for the case of piezoelectric half-space (i.e., infinite piezoelectric layer thickness). The electric displacement and stress intensity factors are defined to characterize the near tip behaviors of the punch. Besides the single punch configuration, the problem of two collinear surface punches is also examined. The paper emphasizes the finite piezoelectric layer thickness. Based on the solutions of finite layer thickness, some new observations, which cannot be found from the solutions of infinite layer thickness, are made.

2. Description of the problem

Consider a piezoelectric ceramic layer shown in Fig. 1. The poling direction of the piezoelectric medium is parallel to the positive y-axis, where (x,y) is the coordinate system. The center of the punch aligns with the y-axis. Suppose the punch is rectangular in shape whose length scale perpendicular to the $(x$-$y)$ plane is considerably larger than its width in the x-direction $(2a)$ and the thickness of the piezoelectric layer in the y-direction (h). For such a specific punch configuration, the problem becomes 2D such that all field variables are functions of x and y only. Denote the displacement along x and y directions as u and v, respectively, and the electric potential as ϕ. Constitutive equations for piezoelectric materials polarized along the y-direction can be expressed as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ D_x \\ D_y \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{13} & 0 & 0 & -e_{31} \\ c_{13} & c_{33} & 0 & 0 & -e_{33} \\ 0 & 0 & c_{44} & -e_{15} & 0 \\ 0 & 0 & e_{15} & \epsilon_{11} & 0 \\ e_{31} & e_{33} & 0 & 0 & \epsilon_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \\ E_x \\ E_y \end{Bmatrix}, \tag{1}$$

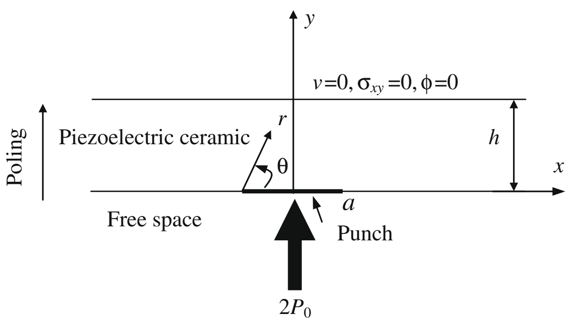

Fig. 1. A piezoelectric layer under a surface rigid punch loaded by a force of $2P_0$.