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The concentration of stress and strain in finite thickness elastic plate containing a circular hole

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Abstract

The elastic stress and strain fields of finite thickness large plate containing a hole are systematically investigated using 3D finite element method. It is found that the stress and strain concentration factors of the finite thickness plate are different even if the plate is in elasticity state except at notch root of plate surface. The maximum stress and strain do not always occur on the mid plane of plate. They occur on the mid plane only in thin plate. The maximum stress and strain concentration factors are not on mid plane and the locations of maximum stress and strain concentration factors are different in thick plate. The maximum stress and strain concentration factors of notch root increase from their plane stress value to their peak values, then decrease gradually with increasing thickness and tend to each constant related to Poisson's ratio of plate, respectively. The stress and strain concentration factors at notch root of plate surface are the same and are the monotonic descent functions of thickness. Their values decrease rapidly and tend to each constant related to Poisson's ratio with plate thickness increasing. The difference between maximum and surface value of stress concentration factor is a monotonic ascent function of thickness. The thicker the plate is or the larger the Poisson's ratio is, the larger the difference is. The corresponding difference of strain concentration factor is similar to the one of stress concentration factor. But the difference magnitude of stress concentration factor is larger than that of strain concentration factor in same plate.

Keywords: Circular hole; Stress concentration factor; Strain concentration factor; Finite thickness

1. Introduction

Cracks in structures often initiate and propagate from the locations of stress or strain concentration. The stress and strain concentration locations are the critical structural details to determine the crack initiation and growth life of engineering structures. Despite careful detail-design, practically many structures contain stress and strain concentrations due to holes. Holes in structural components will create stress or strain concentrations and hence will reduce the mechanical properties. The majority of service cracks nucleate in the area of stress or strain concentration at the edge of a hole. Knowledge of stress and strain concentration in the vicinity

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of a hole should be required for reliable design of structural components. Particularly, the increasing use of high strength materials in the design of structural parts with high mechanical performance requires a better understanding and modeling the behaviour of these structures. Exhaustive stress concentration factor figures and tables have been published by Pilkey (1997) and Young (Young and Budynas, 2002) which account for a wide variety of possible specimen configurations. However, many of these readily available sources of stress concentration factors consider only a two-dimensional theory of elasticity solution.

Experimental evidence shows that for relatively thin plates the crack either originates at the corner, where the hole meets the free surface of the plate, or at the center of the plate. On the other hand for relatively thick plates the crack almost always originates in the vicinity of the corner (Broek, 1974). Evidently, the stress and strain distributions near the edge of a hole are three-dimensional. Knowledge of the three-dimensional stress concentration factor is a prerequisite for an accurate design of structural components. The actual three-dimensional stress and strain fields near a curved boundary are very complex and there are only few analytical three-dimensional solutions available in the literature for non-trivial geometries and particular boundary conditions. By using series expansion and taking finite terms into account, Sternberg et al. (1949) obtained an approximate solution for 3D stress distributions near a circular hole in an infinite plate of arbitrary thickness. Detail analyses for the out-of-plane stress constraint were provided, but for stress concentrations only a brief discussion was given. To develop rigorous analytical solutions a number of approximate theories have been developed to consider the effects of three-dimensional constraint around a stress concentrator (Gregory and Wan, 1988). Many of these theories are based on an asymptotic expansion with respect to a small parameter, which is usually the ratio of the thickness to a characteristic length of the problem. However, underlying assumption limits the validity of any solutions obtained within these theories to only small values of the chosen parameter for thin plates. Analytical as well as numerical investigations reported in the literature show a slight difference between the in-plane stresses obtained from plane-strain theory and these 3D solutions. For example, the increase in the stress concentration factor for an infinite plate with a cylindrical hole subjected to uniaxial loading is less than 3% (Sternberg et al., 1949). In order to obtain a more general solution to the stress concentration problem, Folias and Wang developed a 3D solution using Navier's equation for plates of uniform thickness and with plate faces free of stress (Folias and Wang, 1990). Their results showed the stress concentration factor to be sensitive to the plate thickness and to Poisson's ratio. For thin plate, it was found that the stress concentration factor attains its maximum in the middle of the plate. On the other hand, for thick plate, the stress concentration factor attains its maximum close to the plate surfaces.

Recently, Kotousov and Wang (2002) presented analytical solutions for the three-dimensional stress distribution around typical stress concentrators in an isotropic plate of arbitrary thickness basing on the generalized plane strain theory assumption (Kane and Mindlin, 1956), which assumes that the out-of-plane strain is a constant in the thickness direction. The results were presented on the effects of the plate thickness and Poisson's ratio on the in-plane stress concentration factor and the out-of-plane stress constraint factor. It is shown that the stress concentration factor for a circular hole in an infinite plate is only slightly perturbed from the plane-strain solution over a wide range of thickness to radius ratio. Dealing with stress distributions due to holes, such hypothesis was rejected by Krishnaswamy et al. (1998). With reference to U and V-notches, Li and Guo (2001) also suggested that the assumption constant strain through the thickness can only be used in very thin or very thick plates and this kind of assumption was not suitable near the free surface. The validation of the assumption of generalized plane-strain theory in general stress concentration problems is suspectable. Considering plates of arbitrary thickness containing V-shaped notches, Berto et al. (2004) presented analytical solutions for the three dimensional stress field in the close neighborhood of the stress concentration region by combining Kotousov and Wang's solution (2002) for C_z and Filippi et al.'s solution (2002) for in-plane stresses. The influence of the plate thickness on three-dimensional stress field near notch root was examined by Li et al. (Li et al., 2000; Li and Guo, 2001) using 3D finite element analyses. While keeping the Poisson's ratio constant, Li et al. analyzed the plate thickness influence on the theoretical stress concentration factor, the stress distributions and the out-of-plane stress constraint factor. Bellett et al. (2005) showed experimentally that the common 2D methods for fatigue assessments of isotropic-notched bodies might lead to conservative predictions when applied to three-dimensional geometries. It has been confirmed that the SCF (Stress Concentration Factor) in the interior of the linear elastic isotropic plate with a hole or notch is significantly higher than that on the plate surface or the corresponding planar solutions (Livieri and Nicoletto, 2003). The