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A perturbation method for nonlinear vibrations of imperfect structures: Application to cylindrical shell vibrations

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Abstract

A perturbation method is used to analyse the nonlinear vibration behaviour of imperfect general structures under static preloading. The method is based on a perturbation expansion for both the frequency parameter and the dependent variables. The effects on the linearized and nonlinear vibrations caused by geometric imperfections, a static fundamental state, and a nontrivial static state are included in the perturbation procedure.

The theory is applied in the nonlinear vibration analysis of anisotropic cylindrical shells. In the analysis the specified boundary conditions at the shell edges can be satisfied accurately. The characteristics of the analysis capability are shown through examples of the vibration behaviour of specific shells. Results for single mode and coupled mode nonlinear vibrations of shells are presented. Parametric studies have been performed for a composite shell.

Keywords: Perturbation method; Nonlinear vibration; Shell structures

1. Introduction

Investigations in the field of the nonlinear vibration behaviour of structures used to rely on analytical techniques, before the advent of powerful numerical approaches such as the commonly used Finite Element Method (Evensen, 1974). Although the required computational resources are currently available to use the Finite Element Method in combination with numerical time integration, analytical or semi-analytical (i.e. analytical–numerical) approaches remain indispensable to obtain insight in the behaviour of the structure. The starting point for an analytical approach is generally the set of governing differential equations of the specific structure under consideration. Often the spatial dependence of the solution is taken care of by means of a Galerkin-type discretization, while a perturbation technique is used to describe the temporal behaviour (Nayfeh and Mook, 1979).

As a generalisation of such ad hoc analytical approaches for specific structures, Rehfield (1973) introduced a perturbation method analogous to Koiter's initial postbuckling theory (Koiter, 1945) to investigate the

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Nomenclature

a_s, b_s, b_{ij} nonlinearity coefficients static state
a_d, b_d, b_{ijk} nonlinearity coefficients dynamic state
a_{ijk}, b_{ijkl} nonlinearity coefficients dynamic state, multi-mode analysis
H homogeneous linear functional
L_1 homogeneous linear functional
L_2, L_{11} homogeneous quadratic functional
M generalized mass operator
$\mathbf{q}$ generalized applied load
t time
$\mathbf{u}$ generalized displacement
$\bar{\mathbf{u}}$ generalized geometric imperfection
$\mathbf{u}^*$ geometric imperfection, nontrivial mode
ϵ generalized strain
Λ normalized static load parameter
Λ_c linear bifurcation buckling load
$\boldsymbol{\sigma}$ generalized stress
ξ perturbation parameter
ξ_s, ξ_d, ξ_t perturbation parameter; static, dynamic, imperfect dynamic state
ω radial frequency
ω_c linear natural frequency
ω_{c_I} linear natural frequency, multi-mode analysis

Shell analysis (Section 6)
A_{ij}, B_{ij}, D_{ij} stiffness matrices
$A^*_{ij}, B^*_{ij}, D^*_{ij}$ semi-inverted stiffness matrices
$\bar{A}^*_{ij}, \bar{B}^*_{ij}, \bar{D}^*_{ij}$ nondimensional semi-inverted stiffness matrices
c $= \sqrt{3(1-\nu^2)}$
E reference Young's modulus
E_{11}, E_{22} Young's moduli orthotropic layer
f_0 fundamental state stress function component
f_1, f_2 first-order stress function components
$f_\alpha, f_\beta, f_\gamma$ second-order stress function components
F Airy stress function
$F^{(0)}$, $F^{(1)}$, $F^{(2)}$ stress functions for 0th-, 1st-, and 2nd-order state
G_{12} shear modulus orthotropic layer
h reference shell wall thickness
h_k thickness of k-th layer
L shell length
L_{A^*}, L_{B^*}, L_{D^*} linear operators
L_{NL} nonlinear operator
M_x, M_y, M_{xy}, M_{yx} moment resultants
n number of full waves in circumferential direction
N_x, N_y, N_{xy} stress resultants
N_0 applied axial load ($N_0 = -N_x(x = L)$)
N_{cl} classical buckling load ($N_{cl} = (Eh^2)/(cR)$)
p applied external pressure
$\bar{p}$ normalized external pressure ($\bar{p} = (cR^2)/(Eh^2)p$)
R radius of shell