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Surface and interfacial waves in ionic crystals

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Abstract

The continuum linear theory of ionic crystals is applied to develop a two-dimensional eigenvalue problem in the Stroh formalism. An integral approach is exploited to study the occurrence of surface waves along a free boundary of the crystal. Dispersion relations are obtained by separating real and imaginary parts of the governing system and various boundary conditions are examined. The problem of interfacial waves along the separation boundary between two different crystals is also outlined. Numerical computations are performed for a centrosymmetric crystal (KCl) in order to evaluate bulk wave speeds, limiting speed of surface waves and solutions to the dispersion equations for different boundary conditions.

Keywords: Continuum theory of electroelastic solids; Surface waves; Interfacial waves; Dispersion relations

1. Introduction

Besides its intrinsic interest, continuum models of electromagnetoelastic materials have received attention in the past decades in connection with their application to electroacoustic devices for signal processing, testing of materials, analysis of cracks, etc. The main task of these theories is to provide a suitable set of constitutive equations which account for different material properties such as various electromechanical couplings and anisotropy.

In some instances, continuum models have been supported by microscopic theories based on the study of atomic interactions between material constituents. This is the case of the theory of ionic crystals (see Mindlin, 1968; Maugin, 1988) where the occurrence of polarization gradients in the constitutive assumptions has been justified by a simplified lattice-dynamics theory of alkali halide crystals (Askar et al., 1970; Askar and Lee, 1974). In particular, all the fundamental electromechanical couplings, polarization inertia and dissipative effects have been included in the non-linear theory by Maugin (1988) (see chapter 7) which reduces to the original theory by Mindlin (1968) in the linear case. It is worth remarking that microscopic quantitative methods, based on a quantum first-principle approach, have been recently developed to compute electroelastic proper-

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ties of ionic crystals (King-Smith and Vanderbilt, 1993). They suggest further improvements of the original non-linear continuum theory (Romeo, 2007).

The linear theory has been applied, in the past, to solve various problems both in statics and dynamics (see references in Maugin, 1988). Here we are interested in a systematic approach to the surface and interfacial wave problems. Our aim is to start from the linearized model by Maugin (1988) in order to obtain a two-dimensional eigenvalue problem in the Stroh formalism. We account for a dielectric crystal of arbitrary anisotropy, allowing for polarization inertia in the quasi-static approximation. After having introduced the governing equations in Section 2 we rearrange these in an algebraic system for the amplitudes of plane waves in Section 3. We show that the symmetries of the resulting system are appropriate to develop the integral formalism of Lothe and Barnett (1976a,b) in order to derive the dispersion equation bypassing the solution of the eigenvalue problem.

Suitable forms of the dispersion equations are worked out in Section 4 by separating the real and imaginary parts in the integrated system and exploiting different boundary conditions of mechanical or electric type. Section 5 is devoted to interfacial waves whose compatibility conditions are a consequence of the continuity of the mechanical displacement, the traction and the normal component of the electric displacement.

Numerical evaluations are given in Section 6 for a centrosymmetric crystal including the dispersion relation for bulk waves, the limiting wave speed for surface waves and the speeds of surface modes for different constraints at the surface. The results show that none, one or more surface modes may occur for a given boundary condition at a fixed frequency.

2. Linear equations in the continuum model for ionic crystals

According to Maugin (1988), a set of phenomenological governing equations for an electroelastic polarizable anisotropic crystal, in the quasi-static approximation and using the Heaviside–Lorentz system of units, is given by

$$\begin{aligned} &\rho\ddot{\mathbf{u}} = \nabla\cdot\mathbf{T} + \mathbf{f}, \\ &v\ddot{\mathbf{P}} = \frac{1}{\rho}\nabla\cdot\mathfrak{E} + \mathbf{E} - \nabla\phi, \\ &\nabla\cdot\mathbf{P} - \Delta\phi = 0, \end{aligned} \tag{2.1}$$

at any point in $\mathcal{B}_t$, and

$$\begin{aligned} &\mathbf{nT} = \mathbf{t}^{(\mathbf{n})}, \\ &\frac{1}{\rho}\mathbf{n}\mathfrak{E} = \boldsymbol{\pi}^{(s)} - \mathbf{b}^{(0)}, \\ &\mathbf{n}\cdot\mathbf{P} - \mathbf{n}\cdot\nabla\phi = -\mathbf{n}\cdot\nabla\phi^{(\mathrm{ext})}, \end{aligned} \tag{2.2}$$

at any point on the boundary $\partial\mathcal{B}_t$ with outward normal $\mathbf{n}$. In the previous equations, $\mathbf{u}, \mathbf{P}, \phi$ are, respectively, the mechanical displacement, the polarization and the electric potential, while ρ is the mass density, $\mathbf{f}$ is a density of volume forces acting on $\mathcal{B}_t$ and $v = d_E/\rho$, where d_E is the polarization inertia. The vectors $\mathbf{t}^{(\mathbf{n})}$, $\boldsymbol{\pi}^{(s)}$ and $-\mathbf{b}^{(0)}$ represent, respectively, the mechanical traction acting on the boundary, a surface density of electric dipoles and an intrinsic "polarization traction" due to the presence of the free boundary. In the fully linearized theory the (symmetric) Cauchy stress tensor $\mathbf{T}$, the generalized polarization force $\mathbf{E}$ and the generalized polarization stress $\mathfrak{E}$ are given by the following constitutive equations

$$\begin{aligned} &\mathbf{T} = \mathbb{C}\nabla\mathbf{u} + \mathbb{F}\mathbf{P} + (\nabla\mathbf{P})^{\mathrm{T}}\mathbb{D}, \\ &\mathbf{E} = -\mathbb{A}\mathbf{P} - \mathbb{G}(\nabla\mathbf{P})^{\mathrm{T}} - \nabla\mathbf{u}\mathbb{F}, \\ &\frac{1}{\rho}\mathfrak{E} = \mathbf{P}\mathbb{G} + \mathbb{B}\nabla\mathbf{P} + \mathbb{D}\nabla\mathbf{u}. \end{aligned} \tag{2.3}$$

In these equations, $\mathbb{C}, \mathbb{D}, \mathbb{B}$ are fourth-order tensors which comply with the following symmetry conditions