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Further studies on edge waves in anisotropic elastic plates

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This work is dedicated to the memory of the first author's aunt Lu Jimei

Abstract

A modified Stroh-type formalism for edge waves in unsymmetrical anisotropic plates is derived. Explicit expressions of the fundamental matrices for the formalism are presented. The existence conditions for one or two subsonic edge waves in the unsymmetrical anisotropic plates are discussed based on the formalism, and a procedure for finding an explicit secular equation for the edge-wave speed is proposed.

Keywords: Edge waves; Anisotropic plate; Composite laminates; Stroh formalism; Secular equations

1. Introduction

The subject of flexural edge waves in plates has received increasing attention in recent years due to its potential applications in measurement of material properties and non-destructive evaluation of thin elastic structures, such as aircraft wings, submarine hulls, and floating platforms, etc. Comparing with Rayleigh surface waves on elastic half-spaces, which have been widely studied and have been applied in many engineering areas, the theory of edge waves in plates are still under development, and some in-depth properties are waiting further understanding.

An edge wave in a thin plate is a traveling wave that propagates along the edge of the plate and decays exponentially with the distance from the edge. For symmetric thin plates, the existence of the edge waves was first demonstrated by Konenkov (1960) for isotropic materials, and by Norris (1994) for orthotropic cases. Based on the symmetric plate theories, Thompson et al. (2002) studied the edge waves propagating along non-principal directions, Zakharov and Becker (2003) extended the studies to anisotropic materials,

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Fu (2003) derived an explicit secular equation of the edge waves with a Stroh-like formalism, and Thompson and Abrahams (2005) studied diffraction of flexural waves by cracks in orthotropic thin plates, among others.

In some engineering applications, however, composite laminated structures are required to be modeled as general unsymmetrical anisotropic thin plates with coupled in-plane and out-of-plane deformations. Due to coupling effects, the unsymmetrical plate models are more complicated than the symmetric ones. It seems that the investigation of edge waves in asymmetric anisotropic plates has been reported only by Fu and Brookes (2006) in open literature. In that work, a Stroh-like formalism for the edge waves was derived. Based on the formalism, the existence of the edge waves along the free edge of a semi-infinite unsymmetrical anisotropic thin plate was demonstrated, and a procedure for computing the edge-wave speeds was suggested.

As is known, Stroh formalism based methods have shown some advantages in studying the properties of Rayleigh surface waves in anisotropic elastic materials (Lothe and Barnett, 1976; Chadwick and Smith, 1977; Barnett and Lothe, 1985; Ting, 1996; Ting and Barnett, 1997; Barnett, 2000). More recently, with the method proposed by Taziev (1989) and the fundamental material matrices determined in Stroh formalism, an effective procedure for deriving secular equations of surface waves in general anisotropic materials has also been established (Ting, 2004a,b; Destrade et al., 2005). For the problems of edge waves in plates, most of the reported studies were based on the method similar to Lekhniskii method for static plate problems (Lekhnitskii, 1968), which lead to an algebraic equation for determining eigenvalues. This approach is effective for some cases which are not too complicated, such as uncoupled symmetric plates, isotropic or orthotropic material properties, etc., but may not be suitable for general cases when in-plane and out-of-plane coupled unsymmetrical anisotropic plates are involved. Fu (2003) noted the issues and introduced a Stroh-like formalism into the study of edge waves of symmetric plates. The method was further extended to the case of general unsymmetrical plates to show the existence of at most two edge waves in the structures (Fu and Brookes, 2006).

In the formalism by Fu and Brookes (2006) for the steady edge waves, the in-plane inertia terms in the equations of motion were not considered for simplicities. The formulations and fundamental matrices obtained were also not as compact as expected. In addition, Fu and Brookes (2006) only showed that each unsymmetrical anisotropic plate can support at most two edge waves, but did not elaborate the conditions for the existence of either one or two edge waves.

In this paper, the problems of steady edge waves in general anisotropic laminated plates are revisited by extending the work of Fu and Brookes (2006). An improved Stroh-type formalism is derived by introducing modified generalized stress functions and following the procedure described in Lu (2004). It will be demonstrated that the present formulation will result in similar form of compact and elegant structures and properties as the Stroh formalism for Rayleigh surface waves (Ting, 1996). The fundamental matrices obtained are then simplified, and their elements are expressed explicitly into wave-speed independent and dependent parts. The conditions for existence for one or two subsonic edge waves in the unsymmetrical anisotropic plate are discussed based on the formalism. As an application of the formalism and the fundamental matrices obtained, a method to determine the explicit secular equation for the anisotropic plates is suggested.

2. Basic equations

Consider a Kirchhoff anisotropic laminated plate of thickness h in a Cartesian coordinate system (x_1, x_2, x_3) with $x_3 = 0$ coinciding with the mid-plane of the plate. The constitutive relations of the anisotropic plate with bending extension coupling are given by (Jones, 1999)

$$\begin{Bmatrix} \mathbf{n}_e \\ \mathbf{m}_b \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_e & \mathbf{B}_c \\ \mathbf{B}_c & \mathbf{D}_b \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \end{Bmatrix}, \tag{2.1}$$

where $\mathbf{n}_e$, $\mathbf{m}_b$, $\boldsymbol{\varepsilon}^0$ and $\boldsymbol{\kappa}$ are the stress resultant, the bending moment, the strain and the curvature vectors, respectively, given by

$$\mathbf{n}_e = \begin{Bmatrix} N_{11} \\ N_{22} \\ N_{21} \end{Bmatrix}, \quad \mathbf{m}_b = \begin{Bmatrix} M_{11} \\ M_{22} \\ M_{21} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}^0 = \begin{Bmatrix} \varepsilon^0_{11} \\ \varepsilon^0_{22} \\ 2\varepsilon^0_{21} \end{Bmatrix}, \quad \boldsymbol{\kappa} = \begin{Bmatrix} \kappa_{11} \\ \kappa_{22} \\ 2\kappa_{21} \end{Bmatrix} \tag{2.2}$$