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Consistently linearized constitutive equations of micromechanical models for fibre composites with evolving damage

A. Matzenmiller *, B. Köster

University of Kassel, Institute of Mechanics, D-34109 Kassel, Germany

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Abstract

The numerical analysis of engineering structures is usually based upon the assumptions of a homogeneous as well as a continuous medium. These simplifications are maintained also for structures made of fibre reinforced composite materials which possess by definition a heterogeneous finescale architecture. Furthermore in the course of the loading of such structures void nucleations might arise out of the debonding of the embedded fibres or the growth of microcracks inside the matrix phase. Hence, the assumption of a continuous and homogeneous medium is not valid from a microscopical point of view. Nevertheless, it is numerically advantageous to keep up these simplifying assumptions on the macrolevel. Therefore, the knowledge of the so called macroscopic or effective material behaviour is needed. The overall properties can be described in terms of volume averaged quantities that smear the heterogeneities of the microscopic structure and the influence of its defects. Since the evolution of damage within composite materials means a rather complex process, a purely phenomenological approach is hardly feasible. Hence, the average properties are to be obtained from a micromechanical analysis of the discontinuous and damaged finescale structure. The efficiently reformulated version of the micromechanically based *Generalized Method of Cells* (GMC) provides the macroscopic tangential constitutive tensor in closed-form. The numerical efficiency of the approach allows for the use of the GMC as the constitutive model for nonlinear finite element analyses. Two-scale simulations of macroscale composite structures considering process depending damage evolution on the microscale of heterogeneous media becomes feasible.

Keywords: Composites; Homogenization; Micromechanics; Method of cells; Interface models

1. Introduction

The central objective of this paper is to present a numerically efficient algorithm for obtaining sufficiently reliable predictions of the overall constitutive behaviour of unidirectionally fibre reinforced composite

* Corresponding author.
E-mail address: amat@ifm.maschinenbau.uni-kassel.de (A. Matzenmiller).

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materials. The highly nonlinear, irreversible and anisotropic course of damage processes, as it is typically found for composites, is difficult to be described on a purely phenomenological basis. Therefore, use is made of a computational, micromechanically based approach, originally proposed by Aboudi (1991), which is known as the *Generalized Method of Cells* (GMC). The numerical effort of the GMC was reasonably reduced by Pindera and Bednarcyk (1999) using the degrees of freedom of the discretised microscopic stress field as the basic unknowns instead of those of the microscopic strain field. More accurate resolutions of the microfields of stresses and strains are obtainable by the recently published High-Fidelity-GMC (HFGMC), resting on the principle of asymptotic homogenization, see e.g. Aboudi et al. (2002), Aboudi and Pindera (2004), Bednarcyk et al. (2004) or Bansal and Pindera (2005). While the original GMC only applies piecewise linear functions to approximate the microscopic displacement field, the high fidelity approach applies higher order polynomials. The enhanced accuracy of HFGMC is paid with an appreciable increase of computational effort. Since the nonlinear homogenization procedure has to be carried out for each grid point of numerical integration and at each time step during the finite element analysis of composite structures considering process depending damage evolution, the authors stick to the efficient formulation of the GMC for the time being. An approach similar to the one presented here is pursued by Lissenden (1999), who uses the GMC to predict the nonlinear stress responses of metal matrix composites with interfacial decohesion. The model of Lissenden is portrayed in a more rigorous way introducing an evolving internal variable as a functional of the loading history at the interface. In addition the current paper approximates the softening of the epoxy resin by discrete cracks along predeterminate element boundaries of the cells model.

The central aspect of the paper is to present the tangential, homogeneous constitutive tensor of the composite material in conjunction with softening interface models. The tangential stiffness matrix is deduced analytically from the nonlinear equations of the GMC-model. This special derivation of the homogenized tangential stiffness matrix has not been presented before. The process depending macroscopic stiffness matrix is required, if the GMC approach is used as the constitutive model for the finite element analyses of composite structures.

Within the framework of the reformulated GMC unidirectionally reinforced composites, possessing a periodical microstructure, are considered. Due to this restriction the microstructure can be generated by stringing together a sequence of unit cells, consisting of a single fibre, embedded into the surrounding matrix material, see Fig. 1. The generic unit cell is regarded as a representative volume element (RVE) of the heterogeneous medium. The RVE is subdivided into rectangular subdomains which are referred to as the subcells of the unit cell. The microscopic displacement field within the RVE is then approximated by linear functions within each subcell individually. The stress tensors of the subcells are gained from the local displacement fields and the constitutive tensors of the phases. The continuity of tractions is ensured along all subcell interfaces. Displacement discontinuities are conceded to arise at the common boundaries of neighbouring fibre and matrix subcells in order to model an imperfect bond of the phases. Furthermore, selected interfaces between adjacent

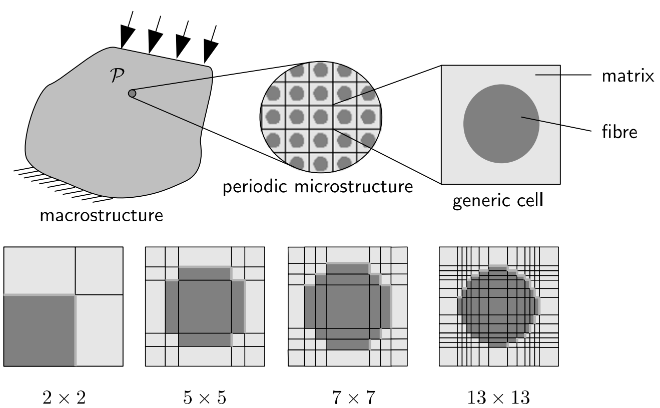

Fig. 1. Top: Macroscopic material point $\mathscr{P}$. Assumed periodical microstructure procreated by a generic cell containing a single fibre. Bottom: Possible discretisations of the generic unit cell by the GMC applying an increasing number of subcells.