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Transient motion of an anisotropic elastic bimaterial due to a line source

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Abstract

The transient motion of an anisotropic elastic bimaterial due to a line force or a line dislocation is studied. The bimaterial is assumed to be at rest and stress-free for $t < 0$. The line source is applied at $t = 0$ and maintained for $t > 0$. A formulation which is an extension to Stroh's formalism for anisotropic elastostatics is employed. The general solution is expressed in terms of the eigenvalues and eigenvectors of a related eigenvalue problem. The method is used to obtain the analytic solutions without the need of performing integral transforms. Numerical examples of the GaAs bimaterial due to a line force or dislocation are presented for illustration.

Keywords: Transient motion; Anisotropic elastic material; Bimaterial; Line source

1. Introduction

Wave propagation in layered elastic media has been a subject of interest in the field of geophysics, acoustics and nondestructive testing. Analysis of the elastodynamic problem is complicated due to the fact that as the propagating wave is interrupted by the interfaces, reflection and transmission waves occur and interfacial waves may also arise. The degree of complexity of the interactions depends on the mechanical properties of the individual layer, number and nature of the interfacial conditions and loading conditions, among other factors. Here we consider the dynamic response of a bimaterial composed of two dissimilar elastic half-spaces of general anisotropy induced by a line force or a line dislocation. The problem serves as a basis for further studies of anisotropic layered systems.

Ma and Huang (1996) considered the problem for an isotropic bimaterial loaded by a line force. The problem is solved by application of Laplace transform method. The inverse transforms are evaluated by means of Cagniard's method. Every and Briggs (1998) presented algorithms based on Fourier transform for calculating the time domain displacement response of fluid-loaded anisotropic half-spaces to impulsive line and point forces at their interface. Wu (2003) have used an extended Stroh's formalism to derive a closed-form solution

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for a suddenly applied interfacial line force or dislocation in an anisotropic bimaterial. In this formulation the solution is expressed in terms of the eigenvalues and eigenvectors of a six-dimensional matrix, which is a function of the material constants, time and position. A major advantage of the formulation is that no integral transforms are required. The fact greatly facilitates derivations of explicit solutions. Recently, Wu and Chen (2006) have further generalized the formulation to treat the problem of a dynamic buried source in a semi-infinite medium. In this paper the generalized formalism proposed by Wu and Chen is used.

The plan of the paper is as follows. In Section 2 the formulation employed is introduced. The problem of a buried line source in an anisotropic bimaterial is studied in Section 3. Numerical examples are given in Section 4. Some conclusions are finally given.

2. Formulation

For two-dimensional deformation in which the Cartesian components of the stress σ_{ij} and the displacement $u_i, i,j = 1,2,3$, are independent of x_3, the equations of motion are

$$\mathbf{t}_{1,1} + \mathbf{t}_{2,2} = \rho\ddot{\mathbf{u}}, \tag{1}$$

where $\mathbf{t}_1 = (\sigma_{11}, \sigma_{21}, \sigma_{31})^{\mathrm{T}}$, $\mathbf{t}_2 = (\sigma_{12}, \sigma_{22}, \sigma_{32})^{\mathrm{T}}$, $\ddot{\mathbf{u}}$ is the acceleration, ρ is the density, a subscript comma denotes partial differentiation with respect to coordinates and overhead dot designates derivative with respect to time t. The stress–strain laws are

$$\mathbf{t}_1 = \mathbf{Q}\mathbf{u}_{,1} + \mathbf{S}\mathbf{u}_{,2}, \tag{2}$$

$$\mathbf{t}_2 = \mathbf{S}^{\mathrm{T}}\mathbf{u}_{,1} + \mathbf{W}\mathbf{u}_{,2}, \tag{3}$$

where the matrices $\mathbf{Q}$, $\mathbf{S}$, and $\mathbf{W}$ are related to elastic constants C_{ijks} by

$$Q_{ik} = C_{i1k1}, \quad S_{ik} = C_{i1k2}, \quad W_{ik} = C_{i2k2}.$$

The equations of motion expressed in terms of displacements are obtained by substituting Eqs. (2) and (3) into Eq. (1) as

$$\mathbf{Q}\mathbf{u}_{,11} + (\mathbf{S} + \mathbf{S}^{\mathrm{T}})\mathbf{u}_{,12} + \mathbf{W}\mathbf{u}_{,22} = \rho\ddot{\mathbf{u}}. \tag{4}$$

Let the displacement be assumed as

$$\mathbf{u}(x_1, x_2, t) = \mathbf{u}(w) \tag{5}$$

with the variable $w(x_1, x_2, t)$ implicitly defined by

$$wt - x_1 - p(w)x_2 - q(w) = 0, \tag{6}$$

where $p(w)$ and $q(w)$ are functions of w.

With Eqs. (5) and (6), Eq. (4) becomes (Wu and Chen, 2006)

$$\frac{\partial w}{\partial x_1}\frac{\partial}{\partial w}\left\{[\mathbf{Q} - \rho w^2\mathbf{I} + p(w)(\mathbf{S} + \mathbf{S}^{\mathrm{T}}) + p(w)^2\mathbf{W}]\frac{\partial w}{\partial x_1}\mathbf{u}'(w)\right\} = \mathbf{0}, \tag{7}$$

where $\mathbf{I}$ is the identity matrix and

$$\frac{\partial w}{\partial x_1} = \frac{1}{t - p'(w)x_2 - q'(w)}. \tag{8}$$

Let $\mathbf{u}'(w)$ be expressed as

$$\mathbf{u}'(w) = f(w)\mathbf{a}(w), \tag{9}$$

where $f(w)$ is an arbitrary scalar function of w. It follows that $\mathbf{u}(w)$ is a solution of Eq. (4) if

$$\mathbf{D}(p, w)\mathbf{a}(w) = \mathbf{0}, \tag{10}$$

where $\mathbf{D}(p, w)$ is given by

$$\mathbf{D}(p, w) = \mathbf{Q} + p(\mathbf{S} + \mathbf{S}^{\mathrm{T}}) + p^2\mathbf{W} - \rho w^2\mathbf{I}. \tag{11}$$