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Coupled damage and plasticity models derived from energy and dissipation potentials

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Abstract

A theoretical framework is defined that allows plasticity and damage models of inelastic behaviour to be combined within a consistent approach. Much emphasis is placed on the fact that, within this framework, the entire constitutive response is specified through two potential functions, with no additional assumptions or evolution equations being necessary. Both plastic strain and damage parameter have roles as internal variables within the theory. Two classes of models are derived: involving respectively uncoupled and coupled plasticity and damage. Examples of application of the theory are presented.

Keywords: Continuum damage mechanics; Elastoplasticity; Hyperelasticity; Internal variables; Hyperplasticity; Geomaterials

1. Introduction

The inelastic behaviour of materials has been successfully modelled using two distinct approaches: plasticity and damage mechanics. Plasticity theory is very widely used in the modelling of ductile metals, and has also been successfully applied to geomaterials. It is based on the concept of additive elastic and plastic strains, the latter only occurring once a yield criterion is satisfied. Many authors have applied thermodynamic principles to plastic materials, and we have had considerable success in applying a method we term "hyperplasticity", which is rooted in thermodynamics, to derive plasticity theories (Houlsby and Puzrin, 2000). Continuum damage mechanics (CDM) was pioneered by Kachanov (1958). The damage of materials is the progressive process by which they break and thus lose strength and stiffness, and this process is represented in CDM by introducing a "damage internal variable". Damage theories are successfully used for modelling materials as diverse as polymers or brittle rocks. Whilst some approaches presented in the literature have a purely phenomenological

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basis, others have based the formulation of CDM taking into account thermodynamic principles. In this paper, we express CDM within the same framework as hyperplasticity, thus encompassing the two concepts of plasticity and damage within a single theory. It is shown that the entire constitutive knowledge of a model that undergoes plasticity and damage can be expressed through definition of two potentials. This allows the constitutive response to be derived directly in a way that ensures consistency with the laws of thermodynamics.

The "hyperplasticity" framework (Houlsby and Puzrin, 2000) allows the development of plasticity theories, within the framework of Generalized Thermodynamics, or Thermodynamics of Internal Variables (TIV), and has much in common with the work of Lubliner (1972), Halphen and Nguyen (1975), Ziegler (1977), and Maugin (1992). The roots of this work are found in that of Ziegler (1977) as developed by Houlsby (1981), Collins and Houlsby (1997), Houlsby and Puzrin (2000) and Einav (2002). A special feature of this approach is an emphasis on the fact that the entire constitutive response of a material can be derived from definition of only two potential functions: an energy potential and a dissipation potential.

It is demonstrated here that the hyperplasticity formulation can be used to develop a damage model, without plasticity. We call this a "damage hyperelastic model". The difference between this type of model and a plasticity model arises from the physical role the internal variables, which in turn derives from the functional nature of the potentials. Pure damage models have been presented by many authors including Krajcinovic (1983), Ortiz (1985), Kattan and Voyiadjis (1990), Maugin (1992) and Lemaitre (1992). For rate independent processes it is customary to assume (effectively) the existence of a yield surface for damage. Here instead the yield surface is derived from the assumed existence of a dissipation potential function, and the evolution of the damage internal variable is defined from further properties of the dissipation potential. In some models the evolution of damage is postulated as a separate evolution law: no such additional assumptions are necessary here.

Many damage models involve the use of an isotropic (scalar) measure of damage. Others have employed two scalar measures of damage for concrete in tension and compression (e.g. Mazars and Pijaudier-Cabot (1989), Fremond and Nedjar (1995), Lee and Fenves (1998), and Nguyen (2005) who used a separate measure of damage for concrete in tension and compression). However, there are good reasons, e.g. based on analysis of microscopic crack distributions, to make use of a tensorial damage variable (e.g., Ladeveze, 1983; Ju, 1989, 1990; Murakami and Kamiya, 1997). Most of those damage-plasticity approaches are based on stress criteria which do not usually give crack orientations in accordance with experimental data at the onset of damage. As a consequence, the direction of damage propagation may not be correct and the advantage of tensorial damage therefore can be lost. Furthermore, the calibration of such models against experimental data is not straightforward, and isotropic damage models are therefore often preferred for routine use. Our purpose here is to explore the combination of damage and plasticity theories, and so we deliberately keep the damage theory as simple as possible and use only isotropic damage.

It is also demonstrated how the concept of multiple surface hyperplasticity, proposed by Puzrin and Houlsby (2001), can allow description of models which undergo damage as well as plasticity (e.g. Lemaitre, 1985; Maugin, 1992; Hansen and Schreyer, 1994; Chaboche, 1997; Li, 1999). We term these "damage hyperplastic" models, and two classes of these are introduced. The first are *uncoupled* damage hyperplastic models in which damage and plasticity are independent processes, although the two processes can (under certain conditions) occur simultaneously. The second class are *coupled* damage hyperplastic models, in which damage and plasticity always occur simultaneously.

Even in the "uncoupled" models described below, plasticity and damage can on occasions occur simultaneously, and are implicitly linked at this stage. Alternative approaches have been made in the past to the coupling between plasticity and damage. Firstly the coupling can be implicitly embedded in the yield and damage criteria (Luccioni et al., 1996; Nguyen and Houlsby, 2004; Salari et al., 2004; Nguyen, 2005), with the material strength being a decreasing function with respect to the damage variable. This implicit coupling characterizes the strength reduction due to the material deterioration and is equivalent to introducing effective instead of nominal stress into the yield function (Lemaitre and Chaboche, 1990; Lemaitre, 1992). This way of introducing coupling enables the constitutive modelling to use separate yield and damage criteria, both of which can be derived from the dissipation function. The corresponding internal variables (damage variable and plastic strains for the coupled model) of the model do not explicitly depend on each other.

An alternative type of coupling has been used by others (Lemaitre, 1985; Lee and Fenves, 1998; Faria et al., 1998), in which only one loading function is specified and used to control the dissipation process. This function