Available online at www.sciencedirect.com
ScienceDirect
International Journal of Solids and Structures 44 (2007) 2509–2522
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Vibration of beams with piezoelectric inclusions

Christian N. Della [1], Dongwei Shu *

School of Mechanical and Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore

Received 16 September 2005; received in revised form 7 June 2006
Available online 8 August 2006

Abstract

A mathematical model for the vibration of beams with piezoelectric inclusions is presented. The piezoelectric inclusion in a non-piezoelectric matrix (host beam) is analyzed as two inhomogeneous inclusion problems, elastic and dielectric, by using Eshelby's equivalent inclusion method. The natural frequency of the beam is determined from the variational principle in Rayleigh quotient form, which is expressed as functions of the elastic strain energy and dielectric energy of the piezoelectric inclusion. The Euler–Bernoulli beam theory and Rayleigh–Ritz approximation technique are used in the present analysis. In addition, a parametric study is conducted to investigate the influence of the energies due to piezoelectric coupling on the natural frequency of the beam.

Keywords: Vibration; Beam; Piezoelectric inclusions; Natural frequency; Eshelby's equivalent inclusion method

1. Introduction

Smart structures are systems that incorporate particular functions such as sensing, processing and actuation. They have the ability to sense certain stimuli and respond in a controlled manner (Chung, 2002). Smart structures are important because of their relevance to structural health monitoring, structural vibration control and transportation engineering. A primary focus in the research of smart structures is the use of piezoelectric materials, since these materials can function both as sensors and actuators (Tani et al., 1998).

A major problem in the dynamic operation of structures is undesirable vibrations (Mackerle, 2003). This is why vibration control and active damping are among the most studied areas using smart materials and structures (Cao et al., 1999; Chee et al., 1998). Extensive research has been done on the vibration control and suppression of structures using piezoelectric materials, as evident from numerous review articles (see for example Ahmadian and DeGuilio, 2001; Irschik, 2002; Rao and Sunar, 1994; Sunar and Rao, 1999; Wetherhold and Aldraihem, 2001). Many mathematical models for laminates and structures with piezoelectric sensors and/or

* Corresponding author. Tel.: +65 67904440; fax: +65 67911859.
E-mail address: mdshu@ntu.edu.sg (D. Shu).
[1] Faculty on leave, Department of Mechanical Engineering, Saint Louis University, Baguio City, Philippines.

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2006.08.002

actuators have been presented in the literature, and reviews of these models have been presented by Alzahrani and Alghamdi (2003), Chee et al. (1998), Gopinathan et al. (2000), Chopra (2002) and Saravanos and Heyliger (1999). Brief summaries on the computational models for composite laminates with piezoelectric sensors and actuators can also be found in Mota Soares et al. (2000) and Reddy (1999). A bibliographical review on the finite element models for the analysis and simulation of smart materials and structures have been presented by Mackerle (2003).

An analysis of these reviews indicates that piezoceramic materials are widely used as sensors and/or actuators. They are either in the form of patches or lamina. The piezoelectric patches are either bonded to or embedded within the structures, whereas the piezoelectric lamina are stacked together with a substrate laminae to form a piezoelectric composite laminate (Chee et al., 1998). However, there are several factors that limit the use of piezoceramic materials, such as their brittle nature and low tensile strength, therefore limiting their ability to conform to curved shapes, and the large add-on mass associated with using typical lead-based piezoceramic (Williams et al., 2002). The use of arrays of piezoelectric sensors and actuators embedded within the structure would remedy the above mentioned restrictions. Due to their small size, these sensors/actuators have the flexibility to conform to curved shapes, and they add little weight to the structure (Badcock and Birt, 2000). In addition, these piezoelectric sensors and actuators can be tailored to achieve a particular smart structure design.

Owing to the small size of the piezoelectric sensors and actuators relative to the size of the host structure, these sensors/actuators can be analyzed as inclusions in a non-piezoelectric matrix (host structure) by using a micromechanics approach. Fan and Qin (1995) analyzed a piezoelectric sensor embedded in a non-piezoelectric elastic matrix by using Eshelby's equivalent inclusion method (Eshelby, 1957; Mura, 1987). The piezoelectric problem was decoupled into an elastic inclusion problem and a dielectric inclusion problem connected by some eigenstrain and eigenelectric field. Jiang et al. (1997, 1999) analyzed the piezoelectric inclusion in a non-piezoelectric matrix by using the Green's function technique.

Krommer and Irschik (1999), Irschik et al. (1998) and Irschik and Ziegler (2001) analyzed the piezoelectric actuation for vibration and shape control of structures as an eigenstrain actuation. An eigenstrain technique was presented by Alghamdi and Dasgupta (1993a,b, 2000, 2001) for the vibration of beams with embedded arrays of piezoelectric sensors and actuators. The embedded sensors and actuators were analyzed as piezoelectric ellipsoidal inclusion in an infinite matrix (host beam) by using Eshelby's equivalent inclusion method. Using the variational principle in Rayleigh quotient form, they formulated an equation for the natural frequency of the beam, which was expressed as functions of the elastic strain energy and dielectric energy of the beam. However, the piezoelectric inclusions were analyzed as elastic inclusions only, thereby neglecting the dielectric effects of the piezoelectric inclusions. The influence of the mechanical–electrical coupling of the piezoelectric sensors on the natural frequency was also neglected in their analyses.

In this research, a mathematical model for the vibration of beams with embedded arrays of piezoelectric sensors and actuators is presented. The piezoelectric sensors and actuators are analyzed as inhomogeneous ellipsoidal inclusions in a non-piezoelectric matrix (host beam) by using Eshelby's equivalent inclusion method (Eshelby, 1957; Mura, 1987). The formulation for the piezoelectric inclusion problem is decoupled into two equivalent inclusion problems, an elastic problem and a dielectric problem. An equation for the natural frequency of the beam is determined using the variational principle in Rayleigh quotient form, which is expressed as functions of the elastic strain energy and dielectric energy of the piezoelectric inclusions. These energies are derived using Mura's formulation for inhomogeneous inclusions. The Euler–Bernoulli beam theory and Rayleigh–Ritz approximation technique are used in the present analysis. In addition, the influences of the energies due to the electromechanical coupling of the actuators and the mechanical–electrical coupling of the sensors on the natural frequency of the beam are studied.

The research is presented as follows. First, the mathematical modeling is presented, which begins with the formulation of the equation for the natural frequency of a piezoelectric body. This is followed the analysis of a non-piezoelectric matrix with piezoelectric ellipsoidal inclusions using Eshelby's equivalent inclusion method. The energies of the piezoelectric inclusions are then formulated, and an explicit solution for the natural frequency of a beam with piezoelectric inclusions is obtained. Next, using the mathematical model presented, the influence of the energies due to piezoelectric coupling on the natural frequency of the beam is studied.