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Elastic stress analysis of rotating converging conical disks subjected to thermal load and having variable density along the radius

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Abstract

An analytical procedure for evaluation of elastic stresses and strains in rotating conical disks, either solid or annular, subjected to thermal load, and having a fictitious density variation along the radius is presented. The procedure is based on two independent integrals of the hypergeometric differential equation describing the displacement field; this procedure is just as general as the one found in technical literature, but less intricate and more reliable. General unpublished relations of stress state and displacement field in conical disk subjected, under elastic conditions, to thermal gradient, and featuring a density variation along the radius are defined. Particular consideration is given to some industrial example of turbine rotors carrying hub and rim with buckets on periphery or radial blades on lateral surfaces. The analytical results obtained by using the new general relations perfectly match those obtained by FEA.

Keywords: Rotating conical disks; Variable thickness; Thermal load; Variable density; Stress analysis

1. Introduction

Rotating conical (or tapered) disks, that are disks whose thickness varies linearly on radius, are widely used in current technical applications (turbine disks, flywheels, gears, pulleys, sprockets, fans, pump impellers and so on). Indeed, they are less expensive than uniform strength disks and they can be designed in order to approximate uniform stress distributions. Therefore, mechanical design of disks involves the evaluation of centrifugal and thermal stresses. Moreover, it is necessary to take into account of stresses due to, for example, radial blades on disk lateral surfaces or buckets along the disk periphery, that can be simulated with fictitious density variation along disk radius.

Analytical close-form solutions of the equilibrium and compatibility equations governing elastic stress and strain state of an arbitrary geometry rotating disk or of complex components, such as a turbine rotor with a hub, web and rim, do not exist in literature. Stress analysis of these disks may be performed using the

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well-known approach formulated by Timoshenko (1956) based on Grammel's method (1923), or numerical techniques, such as the finite difference method (Stanley Thompson and Lester, 1946; Manson, 1947), numerical methods that use truncated Taylor's expansion (Sterner et al., 1994), numerical integration (You et al., 2000), finite element method (see, for example, Fenner, 1987) and boundary element method (Abdul-Mihsein et al., 1985).

Elastic stress and strain theoretical analysis of the rotating converging conical disk, whether solid or annular, axisymmetric and symmetric with respect to the mid-plane, was first carried out by Honegger (1927). Subsequently, Giovannozzi (1950, 1956) generalized Honegger's analysis, by further applying it to the same convergent conical disk subjected to thermal load and featuring a fictitious variation of the density along its radius, as well as to the divergent conical disk, whether solid or annular.

The theoretical approach proposed by Honegger and used by Giovannozzi is based on the formulation originated by Meissner (1913), which transformed the governing homogeneous equation of tapered conical shell subjected to axisymmetric loads into the Gauss-type equation. Afterwards Honegger (1919) further developed the subject and supplied the exact analytical solution of Gauss-type equation. Several years later Honegger (1927) extended this approach to rotating-tapered disks. Unfortunately, the hypergeometric series involved in Honegger's and Giovannozzi's solution of the rotating-tapered disks converge very slowly. For this analytical difficulty, very little works were done and this theoretical methods of approach was practically neglected for years. Only recently this subject has been resumed. About this subject it is relevant to mention works concerning conical disks published by Eraslan et al. (Eraslan and Argeşo, 2002; Orçan and Eraslan, 2002; Eraslan, 2003; Eraslan and Apatay, 2004; Eraslan, 2005; Eraslan and Akis, 2006), concerning the elastic and elastic–plastic analysis of annular rotating disks with variable thickness.

With reference to elastic analysis of conical disks, the authors always use the same procedure, substantially referable to the use of hypergeometric functions. Besides, the articles have at least the following drawbacks: (i) The authors provide the solution of homogeneous equation by means of a linear combination of two hypergeometric functions, the former being the same as the one used in the present work, and the latter being totally different to the one used here: in all the works (Eraslan and Argeşo, 2002; Orçan and Eraslan, 2002; Eraslan, 2003; Eraslan and Apatay, 2004; Eraslan, 2005; Eraslan and Akis, 2006), the second integral, borrowed without critical analysis from Abramowitz and Stegun (1972), is not independent of the first integral; as demonstrated in the present work, its use is not correct. In one work (Eraslan and Argeşo, 2002), the second integral is also obtained from the multiplication of the first integral by an integral function, which is even dependent of the first integral. (ii) The procedure (as the author admit themselves) lacks in generality, because it is not possible to use the analytical approach for solid disks; indeed, both the second integral and the integral function used show a singularity which cannot be integrated at the disk axis, so that the proposed solution is only applicable to annular disks; for solid disks, the solution of the hypergeometric differential equation must be found by numerical integration. (iii) The particular integral of the non-homogeneous differential equation governing the centrifugal field is determined by means of the constant variation method, which still involves the use of incorrect second integral and its first derivative within the Wronskian matrix (Smirnov, 1972); moreover two new integral functions are introduced which, beside making calculation more intricate, blur the physical meaning of this particular solution. (iv) Substantially, the formulation is analytical, but the solution is performed by numerical procedure. (v) Annular disks with rigid inclusion have not physical meaning given that the disk hub is always a deformable structure.

In the present work, the analysis is elastic and the proposed procedure is quite general and valid for rotating conical disks, either convergent or divergent, also subjected to thermal load and having density variation along its radius. As concerns rotating disk having constant density and non subjected to thermal load, only analytical developments concerning the solution of homogeneous differential equation of convergent conical disks are reported; nevertheless the procedure which uses a simple variable change and extends the subject to divergent conical disk is indicated and the correlated final results are reported.

As concerns the solution of the hypergeometric differential equation brought about by the rotating conical disk under elastic loading, it is proposed to directly use its two independent integrals in lieu of the two linear combinations used by Honegger and Giovannozzi, showing the advantages as compared to the latter approach, and validating its results by comparing with those obtained by means of FEA. Moreover this approach is general and it does not present the limits of the procedures found in up-to-date literature.