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Three-dimensional Green's functions for transversely isotropic thermoelastic bimaterials

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ABSTRACT

Green's functions for transversely isotropic thermoelastic biomaterials are established in the paper. We first express the compact general solutions of transversely isotropic thermoelastic material in terms of harmonic functions and introduce six new harmonic functions. The three-dimensional Green's function having a concentrated heat source in steady state is completely solved using these new harmonic functions. The analytical results show some new phenomena of temperature and stress distributions at the interface. The temperature contours are normal to the interface for the isotropic material but not for the orthotropic one. The normal stress contours are parallel to the interface at the boundary in the isotropic region only and shear failure is most likely at the heat source due to the highly degenerated direction of shear stress contours.

1. Introduction

Green's functions play an important role in both applied and theoretical studies on the physics of solids. They are basic building blocks of a lot of further works. Green's functions can be used to construct many analytical solutions of practical engineering problems by superposition and are very important in the boundary element method as well as the study of cracks, defects and inclusions.

For isotropic materials, the Kelvin Green's function is well-known (Banerjee and Butterfield, 1981). For transversely isotropic materials, Lifshitz and Rozentsveig (1947) and Lejcek (1969) derived Green's functions using the Fourier transform method. Elliott (1948), Kroner (1953) and Willis (1965) obtained them using the direct method and Sveklo (1969) found them using the complex method. Pan and Chou (1976) solved Green's function in the form of compact elementary functions. For anisotropic materials, Pan and Yuan (2000) and Pan (2003) obtained the three-dimensional Green's functions for biomaterials with perfect and imperfect interfaces, respectively. The thermal effects are not considered in the above works.

Sharma (1958) studied Green's functions of transversely isotropic thermoelastic materials in integral form. Yu et al. (1992) found the solution for a point heat source in isotropic thermoelastic bimaterials. Berger and Tewary (2001) and Kattis et al. (2004) obtained the two-dimensional Green's functions for anisotropic thermoelastic materials. Chen et al. (2004) derived a compact three-dimensional general solution for transversely isotropic thermoelastic materials. Based on this general solution, Hou et al. (2008) constructed Green's function for infinite and semi-infinite transversely isotropic thermoelastic materials.

As a further extension, the three-dimensional Green's function for a concentrated heat source in a transversely isotropic thermoelastic bimaterial is investigated in this paper. Only steady state is considered. For completeness, the general solution

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of Chen et al. (2004) is summarized in Section 2. In Section 3, six newly found harmonic functions are constructed in terms of elementary functions with undetermined constants. The unique thermoelastic field can be obtained by substituting these functions into the general solutions after determining the constants by the compatibility and equilibrium conditions at interface. Numerical examples are presented in Section 4. The contours of temperature increment and stress components are shown graphically. Finally, the paper is concluded in Section 5.

2. General solutions

We summarize the general solutions of Chen et al. (2004) for use in Section 3. Hou et al. (2008) considered a semi-infinite transversely isotropic thermoelastic material, which is isotropic in the xy-plane. The result will be extended to two such semi-infinite materials joining at the interface $z = 0$ in Section 3.

In the Cartesian coordinate (x, y, z), the constitutive relations are

$$\begin{aligned}
\sigma_x &= c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} - \lambda_{11}\theta, \quad \tau_{yz} = c_{44}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right),\\
\sigma_y &= c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} - \lambda_{11}\theta, \quad \tau_{zx} = c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),\\
\sigma_z &= c_{13}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + c_{33}\frac{\partial w}{\partial z} - \lambda_{33}\theta, \quad \tau_{xy} = c_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),
\end{aligned} \tag{1}$$

where u, v and w are the components of the displacement vector in the x, y and z directions, respectively; σ_{ij} are the stress components and θ is the temperature increment; c_{ij} and λ_{ii} are the elastic and thermal moduli, respectively. $c_{66} = (c_{11} - c_{12})/2$ is held for transversely isotropic thermoelastic materials.

In the absence of body forces, the mechanical equilibrium equations are

$$\begin{aligned}
&\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0,\\
&\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0,\\
&\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0.
\end{aligned} \tag{2a}$$

The heat equilibrium equation is

$$\beta_{11}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + \beta_{33}\frac{\partial^2\theta}{\partial^2 z} = 0, \tag{2b}$$

where β_{ii} $(i = 1, 3)$ are coefficients of heat conduction and $\beta_{22} = \beta_{11}$ when the body is isotropy in the xy-plane.

Chen et al. (2004) gave the general solutions to Eq. (1), (2) as follows:

$$U = \Lambda\left(i\psi_0 + \sum_{j=1}^{3}\psi_j\right), \quad w = \sum_{j=1}^{3} s_j k_{1j}\frac{\partial \psi_j}{\partial z_j}, \quad \theta = k_{23}\frac{\partial^2\psi_3}{\partial z_3^2}, \tag{3a}$$

$$\begin{aligned}
\sigma_1 &= 2\sum_{j=1}^{3}(c_{66} - \omega_j s_j^2)\frac{\partial^2\psi_j}{\partial z_j^2} = -2\sum_{j=1}^{3}(c_{66} - \omega_j s_j^2)\Delta\psi_j,\\
\sigma_2 &= 2c_{66}\Lambda^2\left(i\psi_0 + \sum_{j=1}^{3}\psi_j\right), \quad \sigma_z = \sum_{j=1}^{3}\omega_j\frac{\partial^2\psi_j}{\partial z_j^2} = -\sum_{j=1}^{3}\omega_j\Delta\psi_j,\\
\tau_z &= \Lambda\left(s_0 c_{44} i\frac{\partial\psi_0}{\partial z_0} + \sum_{j=1}^{3}s_j\omega_j\frac{\partial\psi_j}{\partial z_j}\right),
\end{aligned} \tag{3b}$$

where the quantities U, σ_1, σ_2, τ_z can be defined in the Cartesian coordinate (x, y, z) and the cylindrical coordinate (r, ϕ, z) in the complex forms as follows:

$$\begin{aligned}
U &= u + \mathrm{i}v = \mathrm{e}^{\mathrm{i}\phi}(u_r + \mathrm{i}u_\phi),\\
\sigma_1 &= \sigma_x + \sigma_y = \sigma_r + \sigma_\phi,\\
\sigma_2 &= \sigma_x - \sigma_y + 2\mathrm{i}\tau_{xy} = \mathrm{e}^{2\mathrm{i}\phi}(\sigma_r - \sigma_\phi + 2\mathrm{i}\tau_{r\phi}),\\
\tau_z &= \tau_{xz} + \mathrm{i}\tau_{yz} = \mathrm{e}^{\mathrm{i}\phi}(\tau_{zr} + \mathrm{i}\tau_{\phi z}),
\end{aligned} \tag{4}$$

where $\mathrm{i} = \sqrt{-1}$. In additions, for simplicity, we let $z_j = s_j z$ $(j = 0, 1, 2, 3)$, where $s_0 = \sqrt{c_{66}/c_{44}}$, $s_3 = \sqrt{\beta_{11}/\beta_{33}}$ and s_1 and s_2 are the two eigenvalues of the fourth degree equation $a_0 s^4 - b_0\, s^2 + c_0 = 0$ that is Eq. (11) of Chen et al. (2004) in which $a_0 = c_{33}c_{44},\ b_0 = c_{11}c_{33} + c_{44}^2 - (c_{13} + c_{44})^2,\ c_0 = c_{11}c_{44}$. Let ψ_j $(j = 0, 1, 2, 3)$ be the solutions of the following harmonic equations: