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Asymmetric transient response of a hollow cylinder enclosing a compressible fluid

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Abstract

The asymmetric transient response of a hollow cylinder confining a compressible fluid is analyzed. The cylinder is excited by radial displacement prescribed over a rectangular footprint on the cylinder's outer surface. The special case of plane-strain is also analyzed. A comparison of dilatational stress in the solid cylinder and fluid pressure in the fluid-filled cylinder reveals how a projectile may decelerate faster in the latter.

Keywords: Thick cylinder; Wave propagation; Elasto–acoustic interaction; Asymmetric response

1. Introduction

Ballistic experiments on tissue revealed that a projectile might be arrested in tissue confining a fluid compared to the case of solid tissue. Since acoustic impedance of tissue and blood are almost the same, one explanation to this phenomenon is that in solid tissue the stress wave front ahead of the projectile weakens the material by reducing its failure strength. In turn, the projectile penetrates more easily into tissue by tearing and pushing material radially producing a crater that widens and separates from the projectile's surface. This separation reduces the tissue's resistance to penetration. Unlike tissue, the fluid's resistance to projectile motion does not diminish unless it cavitates. However, cavitation is unlikely since fluid pressure increases from confinement. Understanding the stress field ahead of the projectile is necessary to predict how tissue may be weakened from the material science aspects. To study this phenomenon, the model of a hollow cylinder made of gelatin enclosing a compressible fluid like water is considered. This approximates the pericardium and heart confining blood in its cavity.

A comprehensive treatment of the dynamics of solid cylinders can be found in the book by Ewing et al. (1957). Frequency response of a hollow cylinder coupled externally to a confined fluid was treated by Akulenko and Nesterov (1991). Dispersion of time-periodic waves in a fluid wetting a composite hollow cylinder was treated by Xi et al. (2002). Frequency response of a hollow cylinder enclosing a compressible fluid was

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addressed by Ding et al. (1997), and by Chen et al. (2004). These references are limited to axisymmetric frequency response. No references were found addressing asymmetric transient waves in a hollow cylinder enclosing a compressible fluid.

The analysis to follow treats the coupled asymmetric transient response of a hollow cylinder enclosing a compressible fluid. The excitation is a time dependent radial motion prescribed over a rectangular footprint on the cylinder's outer surface. This type of excitation promotes asymmetric motions along the cylinder's surface. The goal is to compare stress wave histories in a solid cylindrical material to those in the fluid confined by a hollow cylindrical material. Density and extensional sound speed in tissue and fluid are approximately the same. The difference is that tissue possesses a small but finite shear stiffness while the fluid relies only on dilatational properties. Although tissue is much weaker in shear than in compression as the ratio of extensional to shear wave speeds is 5, nevertheless shear waves in tissue play a significant role in wave propagation by promoting dispersion. In a fluid, dispersion is caused only by viscous dissipation that is negligible for a fluid like water.

Since the projectile's strength and acoustic impedance are much greater than those of tissue, the excitation transmitted over the outer cylinder boundary at the projectile-tissue interface can be approximated as a given radial time dependent motion in contrast to an unknown pressure excitation. In turn, the excited boundary is mixed meaning that part of the boundary over the footprint has a prescribed motion while the other part is traction-free. The influence method (El-Raheb, 2004) is employed to convert the segment of boundary where motion is prescribed to one where traction is prescribed. The method of static–dynamic superposition (Berry and Naghdi, 1956) is applied to solve for transient response.

Section 2 develops the plane-strain asymmetric model of the infinite cylinder, and Section 3 develops the 3-D asymmetric model of the finite cylinder. Sections 4.1 and 4.2 discuss results of static and dynamic response for the plane-strain and 3-D models. A comparison of histories from the different models explains how pressure in the confined fluid differs from dilatational stress in tissue replacing that fluid.

2. Plane-strain asymmetric model

Consider the plane-strain asymmetric dynamic equilibrium equations in cylindrical coordinates

$$\begin{aligned}&\partial_r\sigma_{rr} + (\sigma_{rr} - \sigma_{\theta\theta})/r + 1/r\partial_\theta\tau_{r\theta} = \rho\partial_{tt}u\\ &\partial_r\tau_{r\theta} + 2\tau_{r\theta}/r + 1/r\partial_\theta\sigma_{\theta\theta} = \rho\partial_{tt}\upsilon\\ &r_i \leqslant r \leqslant r_o, \quad 0 \leqslant \theta \leqslant 2\pi\end{aligned} \tag{1}$$

The convention of stresses and displacements in a cylindrical element is shown in Fig. 1. Boundary conditions are

$$\begin{aligned}&\sigma_{rr}(r_o,\theta;t) = -p_o(H(\theta+\theta_p) - H(\theta-\theta_p))f_p(t), \quad \tau_{r\theta}(r_o,\theta;t) = 0\\ &\sigma_{rr}(r_i,\theta;t) = p_f(r_i,\theta;t), \quad \tau_{r\theta}(r_i,\theta;t) = 0\end{aligned} \tag{2}$$

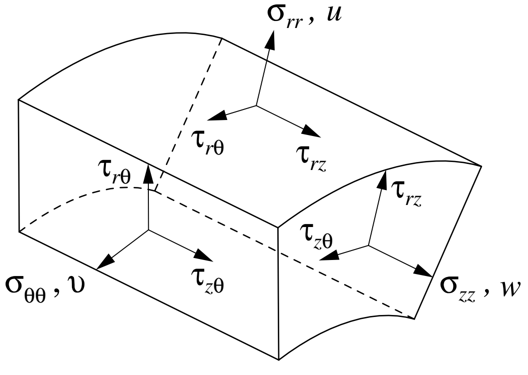

Fig. 1. Cylindrical element with convention of stress and displacement.