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Analytical elasto-creep model of interfacial thermal stresses and strains in trilayer assemblies

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Abstract

A two-dimensional model has been developed for thermal stresses, elastic strains, creep strains, and creep energy density at the interfaces of short and long trilayer assemblies under both plane stress and plane strain conditions. Both linear (viscous) and non-linear creep constitutive behavior under static and cyclic thermal loading can be modeled for all layers. Interfacial stresses and strains are approximated using a combination of exact elasticity solutions and elementary strength of materials theories. Partial differential equations are linearized through a simple finite difference discretization procedure. The approach is mathematically straightforward and can be extended to include plastic behavior and problems involving external loads and a variety of geometries. The model can provide input data for thermal fatigue life prediction in solder or adhesive joints. For a typical solder joint, it is demonstrated that the predicted cyclic stress–strain hysteresis shows shakedown and a rapid stabilization of the creep energy dissipation per cycle in agreement with the predictions of finite element analysis.

Keywords: Trilayer; Layered beam; Laminated; Interfacial stress; Creep; Viscoelastic; Strain energy; Thermal cycling; Hysteresis; Ratcheting; Shakedown; Thermal stress; Thermal strain; Adhesive bond; Solder joint; Stress singularity

1. Introduction

Layered assemblies of dissimilar materials are a common feature of laminated structures, coatings and joints that are soldered, brazed or adhesively bonded. Under thermal loading such trilayer assemblies can suffer from unacceptable deformation (e.g. Madras et al., 1996), delamination and cracking because of expansivity and rigidity mismatch (e.g. Wang et al., 2000), residual stresses (e.g. Humfeld and Dillard, 1998), and creep-fatigue damage in structures with viscoelastic or viscoplastic materials (e.g. Qi et al., 2006).

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Interfacial thermal stress and strain distributions in layered structures have been analyzed using three main approaches: two-dimensional elasticity solutions, finite element (FE) analysis, and elementary beam theory. Many references can be found in the recent papers of Shen and Suresh (1996), Suhir (2001), Wen and Basaran (2004), and Ghorbani and Spelt (2005).

Two-dimensional elasticity solutions for interfacial thermal stresses were developed by Hayashi (1967), Bogy (1968), Zeyfang (1971), Chen et al. (1982), Yin (1991), Xie and Sitaraman (2000), Chen et al. (2003), Matsunaga (2004), and many others. This method usually leads to differential equations which must be solved numerically. It also results in an unrealistic singular stress field at the free edges of interfaces if an exact solution with strict adherence to the constitutive relations of linear elasticity is sought. In reality, the exact elasticity solution around free edges reflects the existence of a region with an intense stress or stress gradient (Yin, 1991).

Finite element analysis (FEA) has received wide attention in the analysis of layered assemblies. For instance, Mackerle (2002) gives a bibliography of 867 FEA papers in adhesive bonding, soldering, and brazing published in the period 1996–2002. Nevertheless, as shown by Glaser (1990), Basaran and Zhao (2001), Ghorbani and Spelt (2005), and many others, the predictions of elastic FEA are strongly mesh sensitive around the free corners of interlayers under thermal loading. This is a consequence of the underlying elasticity solutions which predict that peel stresses approach infinity at the free edges. Through the equilibrium equations this also affects the other stresses (Yin, 1991), causing elastic FE models to be accurate only away from the free edges.

The present elasto-creep analysis is based on the structural mechanics model of Ghorbani and Spelt (2005) for the thermal stresses in long and short trilayer assemblies. This model satisfies all equilibrium and compatibility requirements using compliances defined through elasticity solutions for both plane stress and plane strain conditions. The governing differential equations were solved using a straightforward finite difference procedure.

A few analytical models have been presented for the inelastic analysis of layered structures. Suhir (1986) proposed a closed-form structural mechanics solution for the elasto-plastic interfacial shear stress distribution in bilayer assemblies based on his structural mechanics approach and deformational plasticity. In this model, the plastic deformations due to other stress components (i.e. peel, axial, and out-of-plane) were neglected and equilibrium requirements for peel stresses were not met. For materials such as solder and polymeric adhesives, creep can be more detrimental than plastic deformation (Akay et al., 1997). Mirman and Knecht (1990) extended Suhir's model to account for creep strains due to only shear stresses in elongated bonded layers. However, the model neglected other stresses and creep deformations in trilayers.

Shen and Suresh (1995, 1996) developed analytical models for elasto-plastic and steady-state creep deformations in multi-layered materials during thermal cycling. The models were aimed at capturing residual stresses, stress relaxation and curvature reversal during monotonic temperature change. It was reported that the elasto-plastic model provided a better match with the experimental results during the heating phase, whereas the creep model was better during the cooling phase. It was assumed that stresses and strains varied only in the through-thickness direction (independent of longitudinal position), with the curvature being the same for all layers. This is an approximation since the radius of curvature is variable in both the longitudinal and transverse directions from one layer to another (Ghorbani and Spelt, 2005).

Madras et al. (1996), in an attempt to explain the permanent deformations observed experimentally in adhesively bonded optical coatings during thermal cycling, modeled the adhesive using a simple Maxwell (spring-dashpot) element with a temperature-dependent viscosity. Humfeld and Dillard (1998) developed an analytical Maxwell-type model to calculate the residual stresses and stress relaxation in polymeric materials bonding to stiff elastic substrates subject to thermal cycling. The model treats the interlayer as a bulk Maxwell element (i.e. a spring and a dashpot in series), assuming that the axial stress is the only component of stress in the interlayer and that it is constant. The model is therefore applicable to the midpoint of the adhesive layer, where the interfacial stresses vanish. Using FEA, Dillard et al. (2003) provided further insights into shrinkage and hysteresis in viscoelastic adhesive joints under cyclic thermal loads.

As mentioned above, the present 2D elasto-creep analytical model for trilayer assemblies extends an earlier elastic model for interfacial thermal stresses (Ghorbani and Spelt, 2005). The model is applicable to trilayers with any aspect ratio (i.e. both long and short) under either plane stress or plane strain conditions, and can be used with either a linear (viscous) or non-linear creep model for any of the three layers. Sample calculations