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A decoupled modal analysis for nonlinear dynamics of an orthotropic thin laminate in a simply supported boundary condition subject to thermal mechanical loading

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Abstract

In this paper, we analyze the nonlinear dynamic response of an orthotropic laminate in a simply supported boundary condition subject to thermal and mechanical loading. The equation of motion for the laminate's deflection is obtained in a decoupled Duffing equation by means of a Galerkin-type method without Berger's approximations. The Duffing equation incorporates an arbitrary thermal field, with both the in-plane and transverse temperature variations in a steady-state and a transient state. The formulation indicates that the transverse temperature variation produces an additional pressure load, while the in-plane temperature variation affects the system frequency. The equation allows for characterization of the laminate behaviors in nonlinear thermal buckling, thermal vibration and thermal mechanical response.

Keywords: Laminate nonlinear dynamics; Nonlinear thermal buckling and vibration; Chaos; Duffing equation

1. Introduction

Thermal field with a temperature variation can cause the nonlinear deformation of a thin laminated structure, similar to a mechanical load does. This is particularly true for a thin laminate used as a micro-electro-mechanical structure (MEMS), such as a circuit board that is in a dynamic motion subject to a thermal electric field. The MEMS structure is usually composed of conduction and insulation layers; as such, the electro-thermal coupling effect generates non-uniform temperatures in each lamina plane and through the laminate thickness. The deformation and stresses of each lamina differ from one another due to the differences in their thermal and mechanical properties.

The nonlinear analysis of a laminate subject to mechanical loading has been based on the governing equations of motion developed by Whitney and Leissa with nonlinear strain fields (Whitney and Leissa, 1969). Extensive investigation on the laminate nonlinear vibration and buckling behaviors has been completed (Chia,

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Nomenclature

A	stiffness matrix of the laminate
A^*	inverse matrix of A
A^*_{ij}	elements of A^*
B	coupling rigidity matrix of the laminate
C	stiffness matrix of the laminate for constitutive equation of stress and strain
C_0	constant
C_{ij}	coefficient of the stiffness matrix
$E^{(k)}$	kth lamina Young's Modulus
$F(x,y,t)$	Airy's stress function
$\widetilde{F}(x,y,t)$	Airy's stress function with complementary function
$\tilde{I}$, I, I_1, I_2	inertia of the laminate
M	applied moment vector
M^{T}	thermal moment vector
$N(w)$	function operator for in-plane forces and deflection
N	in-plane force vector
N^{T}	in-plane thermal force vector
Q_{mn}	external pressure load on the laminate
$T_0(x,y)$	in-plane temperature variation
$T_1(x,y)$	temperature variation through the thickness of the laminate
T^0_{mn}, T^1_{mn}	Fourier series coefficients for $T_0(x,y)$, $T_1(x,y)$ and $T_c(x,y)$, respectively
a	laminate length in x
b	laminate length in y
$l_j(\vec{x}_0)$	Lyapunov exponent
m^*, p^*, r^*, s^*	stiffness coefficients
q^{T}	transverse thermal load
q	Duffing equation forcing function due to pressure load
$u(y,y)$	in-plane deformation of laminate in x direction
$v(x,y)$	in-plane deformation of laminate in y direction
$v^{(1)}$, $v^{(2)}$	elements of Lyapunov exponent
$w(x,y,t)$	transverse deflection function
$\ddot{w}(x,y,t)$	transverse deflection acceleration function
$\hat{w}(x,y,t)$	transverse deflection approximating function
$\lvert W(t)\rvert$, $\lvert\ddot{W}(t)\rvert$	transverse deflection and acceleration, respectively
$\lvert W_0\rvert$	finite deflection
$\alpha^{(k)}_x$, $\alpha^{(k)}_y$, $\alpha^{(k)}_{xy}$	kth lamina coefficient of thermal expansion
α_m	mth mode dimension in x
β_n	nth mode dimension in y
ε_i, ε_{ij}, γ_{ij}	strain components
e_i	strain invariants
κ	thermal conductivity
$\hat{\gamma}$	geometric aspect ratio
γ_{mn}	aspect ratio
$\rho^{(k)}_i$	kth lamina material density
η	coefficient for thermal force N^{T}
ξ	coefficient for thermal moment M^{T}
ω	excitation frequency
ω_{0_mn}	natural frequency
ω_{mn}	system frequency with thermal effect