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An exact solution for the three-phase thermo-electro-magneto-elastic cylinder model and its application to piezoelectric–magnetic fiber composites

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ABSTRACT

A three-phase cylindrical model for analyzing fiber composite subject to in-plane mechanical load under the coupling effects of multiple physical fields (thermo, electric, magnetic and elastic) is presented. By introducing an eigenstrain corresponding to the thermo-electro-magnetic-elastic effect, the complex multi-field coupling problem can be reduced to a formal in-plane elasticity problem for which an exact closed form solution is available. The present three-phase model can be applied to fiber/interphase/matrix composites, such that a lot of interesting thermo-electro-magnetism and stress coupling phenomena induced by the interphase layer are revealed. The present model can also be applied to fiber/matrix composites, in terms of which a generalized self-consistent method (GSCM) is developed for predicting the effective properties of piezoelectric–magnetic fiber reinforced composites. The effective piezoelectric, piezomagnetic, thermoelectric and magnetoelectric moduli can be expressed in compact explicit formulae for direct references and applications. A comparison of the predictions by the GSCM with available experimental data is presented, and interesting magnification effects and peculiar product properties are discussed. As a theoretical basis for the GSCM, the equivalence of the three sets of different average field equations in predicting the effective properties are proved, and this fact provides a strong evidence of mathematical rigor and physical realism in the formulation.

1. Introduction

Functional composite materials, such as piezoelectric, magnetostrictive and thermoelectroelastic composites have been rapidly developing with increasing applications in ultrasonic imaging devices, sensors, actuators and transducers etc. Such composites inherit the characteristics of functional materials, such as the piezoelectric and piezomagnetic properties which can be tailored to meet specific applications.

Some composite materials can provide superior properties compared to their mother monolithic constituent materials. Smith et al. (1985) and Shaulov et al. (1989) found that the piezocomposites can provide a higher piezoelectric strain modulus d_{31} than the constituents; Dunn (1993) numerical results showed that the effective thermal expansion coefficients of composites could significantly exceed those of the matrix and the fiber phases. An important application of composite structure is the use of the product property, which is found in the composite structures but is absent in the individual phases (Ryu et al., 2002). Van Suchtelen (1972) suggested that the combination of piezoelectric–piezomagnetic phases may exhibit a new

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material property – the magnetoelectric (ME) coupling effect (the product property). Van Run et al. (1974) reported that the fabrication of $BaTiO_3$–$CoFe_2O_4$ composite has a ME coefficient hundred times higher than that of Cr_2O_3 which was then one of the single-phase materials of the highest ME coefficient.

Inspired by the above interesting and exciting multi-field coupling phenomena as well as the excellent designability of composites including piezoelectric–magnetic fiber composites, scientists and engineers begin to pursue the optimal design for the desired applications. With the rapid advancement in technological research, it is expected that composites and completely novel materials could be conveniently designed and manufactured by direct engineering of their constituents. The traditional phenomenological approach, however, is of limited use in making prediction on the behaviours of new composite materials which have not come to exist yet. It requires that researches go beyond such phenomenology for a more comprehensive understanding of the interaction of microstructures and their thermo-electro-magneto-elastic coupling properties (Tadmor et al., 2000).

Micromechanics methods (for example, Hori and Nemat-Nasser, 1998) are useful tools to predict effective properties of composites. The dilute (Eshelby, 1957), self-consistent (Budiansky, 1965; Hill, 1965), differential (Mclaughlin, 1977) and Mori–Tanaka methods (Mori and Tanaka, 1973; Benveniste, 1987) are based on the two-phase micromechanics models that have been extensively used. In order to extend the two-phase model to the multi-field coupled composite structure, the coupled Eshelby's tensor analogs to the elasticity has been developed. Deeg (1980) and Wang (1992) presented the tensor solution of a piezoelectric ellipsoidal inclusion embedded in an infinite medium. Dunn and Taya (1993) simplified the piezoelectric Eshelby's tensors of the elliptic fiber problem and put them in explicit form instead of the elliptic integrals, and then they extended the dilute, self-consistent, Mori–Tanaka and differential micromechanics methods to cover piezoelectric composites. As for the piezomagnetic materials, Nan (1994, 1997) proposed a model to determine the coupled ME effect and coupled local fields of composites based on the Greens' function method and perturbation theory developed for piezoelectric composites. Huang and Kuo (1997) directly extended the Eshelby type equivalent inclusion method to piezomagnetic composites which is originally developed for piezoelectric composites by Dunn and Taya (1993), and they proposal the analogous simplification on Eshelby's tensors (Huang, 1998). The thermal expansion is another important coupled field problem. Similarly, many researchers have extended the classical two-phase models to thermoelectroelastic composites (Dunn, 1993; Budiansky, 1965, etc.). Using the method of effective field, Sevostianov et al. (2001) derived the electroelastic constants of piezocomposites, and Levin and Luchaninov (2001) further considered the thermo-piezoelectric matrix composites. They presented explicit expressions for the effective pyroelectric, dielectric, piezoelectric, elastic and thermoelastic constants of such composites.

The generalized self-consistent method (GSCM) is a more sophisticated micromechanics approach, which is based on the three-phase model of inclusion/matrix/composite. The method was originally developed by Kerner (1956) and could be regarded as an extension of the composite cylinder/sphere model (Hashin, 1962; Hashin and Rosen, 1964). The boundary conditions of the GSCM model are simpler and more reasonable compared to the composite cylinder/sphere model. However, the GSCM may increase the complexity of the solution process, and Smith (1974) and Christensen and Lo (1979) took a long journey to arrive at the correct solutions. It is shown that the GSCM possesses the mathematical rigor in their elasticity formulation and physical realism (Christensen, 1998). Comparisons between the GSCM and other micromechanics methods also reveal that the GSCM predictions exhibit the best agreement with experiment results (Christensen, 1990). Luo and Weng (1987) proposed a modified Mori–Tanaka method which is based on the three-phase model instead of the Eshelby's problem. As for thermo-electro-magneto-elastic composites, Grekov et al. (1989) studied the composite cylinder model for piezocomposites, and Benveniste (1995) gave the predictions of the ME coefficient, which is based on the composite cylinder model and the theory of exact connections of multi-fields. Jiang and Cheung (2001) presented a three-phase piezoelectric cylinder model. The model was applied to fiber/interphase/matrix composites, whereby the effect of the interphase layer was studied. Jiang et al. (2001) also presented a three-phase confocal elliptical model, in which the GSCM for piezocomposites was developed, and Sudak (2003) studied the effect of an interphase layer on the electroelastic stresses within a three-phase elliptic inclusion by using this model. To our best knowledge, the researches on the three-phase model are limited to a simple case of antiplane shear coupled with inplane electric load. So far, no report has been found for a three-phase model under inplane mechanical load coupling with thermo-electro-magnetical loads. For such a problem, multi-field coupling leads to a set of rather complicated differential equations. To overcome the mathematical difficulties, in this paper, an eigenstrain corresponding to the thermo-electro-magnetic fields is introduced. As a result, the multi-field coupled problem is reduced to an equivalent inplane elasticity problem, for which the mathematical manipulation is greatly simplified and a compact solution in closed form is available.

This paper is organized as follows. Section 2 establishes the three-phase model and introduces the eigenstrain to simplify the constitutive equations. Section 3 provides a solution in closed form for the elasticity inplane problem with an eigenstrain by the complex variable method. Section 4 deals with stress concentrations under thermo-electro-magneto-mechanical coupling loads. The full thermo-electro-magneto-elastic moduli are obtained in compact explicit form in Section 5, and through numerical examples, interesting coupling phenomena such as the abnormal magnification effect and the product effect for ME composites are discussed. In the appendix, the equivalence of three sets of different average field equations in predicting the effective properties by the GSCM is proved. They are: (1) the averaged stress in the representative volume element (RVE) is equal to the far-field one; (2) the averaged strain in the RVE is equal to the far-field one; (3) the averaged stress and strain in RVE satisfy the stress-strain relationship of the composite with the as-yet unknown effective properties.