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Dispersion relations for SH wave in magneto-electro-elastic heterostructures

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ABSTRACT

In the present work the dispersion relations of stationary SH waves in a heteroestructure with magneto-electro-elastic properties have been obtained. The calculations were done taking into consideration the symmetry of the system and separating the solutions in symmetric and anti-symmetric parts. Different limit cases are presented. The dispersion curves and amplitudes of vibration are shown for different configurations.

1. Introduction

The development of smart structures is currently receiving widespread attention owing to potential applications in several branches of engineering e.g. integrated control architecture with highly distributed sensors and actuators. More recent advances are the smart or intelligent materials where piezoelectric and piezomagnetic properties are involved. These composite materials can take the advantages of each constituent and consequently have superior coupling magneto-electric effects as compared to conventional piezoelectric or piezomagnetic materials. The magneto-electric coupling effect is a new product property of the composite, since it is absent in each constituent (Harshe et al., 1993; Avellaneda and Harshe, 1994; Benveniste, 1995). In some cases these materials can be considered in a single magneto-electro-elastic phase (Nam, 1994; Li and Dunn, 1998; Aboudi, 2000; Chen et al., 2002) and can find applications in diverse branches of the modern engineering. Consequently, they are extensively used as magnetic field probes in electric packaging, acoustics, hydrophones, medical ultrasonic imaging, sensors, and actuators with the responsibility of magneto-electro-mechanical energy conversion. The development of piezoelectric/piezomagnetic composites has its roots in the early work of Suchtelen (1972) who proposed that the combination of piezoelectric–piezomagnetic phases may exhibit a new material property-the magneto-electric coupling effect. More detailed information is given in Fiebig (2005).

The problem of wave behavior in this type of materials has been studied by different authors in different geometries. Alshits et al. (1994) studied the existence of localized acoustic waves on the interface between two piezocrystals of arbi-

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trary anisotropy. Pan (2001) derived an exact closed-form solution for the static deformation of the multilayered piezoelectric and piezomagnetic plates based on the quasi-Stroh formalism and the propagator matrix method. Pan and Heyliger (2002) extended the analytical method of Pan (2001) to the free vibration three-dimensional, linear anisotropic, magneto-electro-elastic, simply supported, and multilayered rectangular plates. Wang et al. (2003) derived the state vector equations for three-dimensional, orthotropic and linearly magneto-electro-elastic media and solution of theses equations is based on the mixed formulation, in which basic unknowns are formed by collecting not only displacements, electrical potential and magnetic potential but also some of stress, electric displacements and magnetic induction. On the other hand, Feng et al. (2006) investigated the scattering of SH waves by a magneto-electro-elastic cylindrical inclusion. Recently, Chen et al. (2007) presented an analytical treatment for the propagation of harmonic waves in infinitely extended, magneto-electro-elastic multilayered plates based on the state vector approach and Du et al. (2007) studied an exact approach to investigate Love waves in a piezomagnetic material layer bonded to a semi-infinite piezoelectric substrate with the consideration of initial stress.

The main goal of this work is related to the study of the behavior of stationary SH waves in a hetero-structure with magneto-electro-elastic materials. Confined modes in this hetero-structure made of magneto-electro-elastic materials are considered. The surface waves at the interface of the materials are taken into consideration. The governing system of partial differential equations is solved using the symmetry of the system and considering the superficial acoustic wave. Solutions are separated in symmetric and anti-symmetric states. The dispersion relations are derived for symmetric and anti symmetric states. Different limit cases are presented. Confined modes for different configurations of hetero-structure are studied and other configurations are pointed out where the confined modes are not observed. The dispersion curves and amplitudes of vibration are shown for some cases.

2. Fundamental equations for magneto-electro-elastic homogeneous materials

We consider a material who exhibits magneto-electro-elastic properties with 6 mm symmetry where magnetization and polarization are in the z axis direction. The constitutive equations can be expressed in matrix form as

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ D_1 \\ D_2 \\ D_3 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 & 0 & 0 & -e_{31} & 0 & 0 & -f_{31} \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 & 0 & 0 & -e_{31} & 0 & 0 & -f_{31} \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 & 0 & 0 & -e_{33} & 0 & 0 & -f_{33} \\ 0 & 0 & 0 & c_{44} & 0 & 0 & 0 & -e_{15} & 0 & 0 & -f_{15} & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 & -e_{15} & 0 & 0 & -f_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15} & 0 & \varepsilon_{11} & 0 & 0 & g_{11} & 0 & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 & 0 & \varepsilon_{11} & 0 & 0 & g_{11} & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 & 0 & 0 & \varepsilon_{33} & 0 & 0 & g_{33} \\ 0 & 0 & 0 & 0 & f_{15} & 0 & g_{11} & 0 & 0 & \mu_{11} & 0 & 0 \\ 0 & 0 & 0 & f_{15} & 0 & 0 & 0 & g_{11} & 0 & 0 & \mu_{11} & 0 \\ f_{31} & f_{31} & f_{33} & 0 & 0 & 0 & 0 & 0 & g_{33} & 0 & 0 & \mu_{33} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ E_1 \\ E_2 \\ E_3 \\ H_1 \\ H_2 \\ H_3 \end{bmatrix} \tag{1}$$

where $c_{66} = (c_{11} - c_{12})/2$ and T_p, $S_p(p, q = 1,\cdots,6)$ are the components of the stress and strain second order tensors, respectively, written in reduced notation; D_i,E_i,B_i and H_i (i = 1, 2, 3) are the component of the electric displacement, electric field, magnetic induction and magnetic field, respectively. On the other hand, c_{pq}, ε_{ij}, μ_{ij}, e_{iq}, f_{iq} and $g_{ij}(p, q = 1,\ldots,6;\ i, j = 1, 2, 3)$ are the elastic, dielectric, magnetic permeability, piezoelectric, piezomagnetic and magnetoelectric coefficients, respectively.

The strain-displacement equations are

$$\begin{aligned} S_1 &= \frac{\partial u_x}{\partial x}, \quad S_4 = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}, \\ S_2 &= \frac{\partial u_y}{\partial y}, \quad S_5 = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}, \\ S_3 &= \frac{\partial u_z}{\partial z}, \quad S_6 = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}. \end{aligned} \tag{2}$$

where u_x, u_y, u_z are the elastic displacement components.

Under the quasi-static approximation, the electric and magnetic fields can be expressed as the gradient of two scalar potentials functions, i.e. the electric potential function $\varphi(x,y,z,t)$ and the magnetic potential function $\psi(x,y,z,t)$,

$$E_1 = -\frac{\partial \varphi}{\partial x}, \quad E_2 = -\frac{\partial \varphi}{\partial y}, \quad E_3 = -\frac{\partial \varphi}{\partial z}, \tag{3}$$

$$H_1 = -\frac{\partial \psi}{\partial x}, \quad H_2 = -\frac{\partial \psi}{\partial y}, \quad H_3 = -\frac{\partial \psi}{\partial z}. \tag{4}$$