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Confinement-sensitive plasticity constitutive model for concrete in triaxial compression

Vassilis K. Papanikolaou *, Andreas J. Kappos

Laboratory of Reinforced Concrete and Masonry Structures, Civil Engineering Department, Aristotle University of Thessaloniki, P.O. Box 482, Thessaloniki 54124, Greece

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Abstract

In this paper, a confinement-sensitive plasticity constitutive model for concrete in triaxial compression is presented, aiming to describe the strength and deformational behaviour of both normal and high-strength concrete under multiaxial compression. It incorporates a three-parameter loading surface, uncoupled hardening and softening functions following the accumulation of plastic volumetric strain and a nonlinear Lode-angle dependent plastic potential function. The various model parameters are calibrated mainly on the basis of a large experimental database and are expressed in terms of only the uniaxial compressive concrete strength, leading to a single-parameter model, suitable for practical applications. The model's performance is evaluated against experimental results and it is found that both the increased strength and deformation capacity of confined concrete are properly captured.

Keywords: Concrete; Confinement; Plasticity; Constitutive modelling; Triaxial compression; Deformation capacity; 3D finite elements

1. Introduction

Three-dimensional finite element analysis of confined concrete members such as columns and bridge piers of arbitrary section (Papanikolaou and Kappos, 2005) requires sophisticated constitutive models, capable of describing the increased strength and deformation capacity of concrete under multiaxial compressive stress states. Different theories and formulations for constitutive modelling of concrete have been suggested in the literature in the past, characterised by a variable degree of complexity, ranging from phenomenological elastic nonlinear (e.g. Darwin and Pecknold, 1977) to complex endochronic plasticity and microplane models (e.g. Bažant and Prat, 1988). The degree of success for each analytical approach depends on the balance between accuracy and practicality, with the latter often hindered by numerous model parameters, often with blurred physical meaning and hence difficult to calibrate against experimental evidence. In the present study, the target is twofold: the formulation of a concrete constitutive model that successfully simulates the basic aspects of

* Corresponding author. Tel.: +30 2310995662; fax: +30 2310995614.
E-mail address: billy@ee.auth.gr (V.K. Papanikolaou).

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compressive behaviour in the presence of confinement, and an accompanying calibration scheme based on the minimum possible number of material parameters.

The proposed concrete constitutive model follows the classical theory of incremental plasticity (e.g. Chen and Han, 1988). The main components of the model are: a loading surface appropriate for cementitious materials, hardening and softening functions describing the evolution of the loading surface during plastic flow and an appropriate combination between a hardening/softening parameter and a plastic potential function for correctly estimating the deformation capacity of concrete under triaxial compression. The mathematical description of the above components is presented in the subsequent sections, each one followed by a calibration procedure mainly based on experimental results from the literature. The ensuing values for the various model parameters eventually depend only on the mean uniaxial compressive concrete strength (f_c), which renders the constitutive model to a single-parameter one and hence more convenient for practical applications than multi-parameter models.

It should be noted that since the suggested model is applicable in the concrete compression regime only, it should be properly combined with a tensile fracture model for it to be usable in general finite element applications, where tensile stresses can not be excluded. This combination can be realised by treating plastic and fracture strain separately and apply an iterative scheme to preserve stress equivalence (e.g. De Borst, 1986; Červenka et al., 1998). The development of a complete model and its application to finite element analysis of confined reinforced concrete members is currently under investigation by the writers. Furthermore, the localization of deformations under triaxial compression (Van Mier, 1986) is not currently handled by the current constitutive model; this may introduce mesh dependency in certain applications. However, the softening function can be extended in future studies to either account for the fracture energy of concrete in compression or be associated with deformations instead of strains.

2. Fundamental constitutive equations and definition of the principal stress space

According to the classical theory of plasticity, the incremental total strain vector ($\mathrm{d}\boldsymbol{\varepsilon}$) is decomposed into an elastic ($\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{e}}$) and a plastic ($\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}$) component:

$$\mathrm{d}\boldsymbol{\varepsilon} = \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{e}} + \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} \tag{1}$$

The reversible elastic strain increments are related to the stress increments through a Hook-type elasticity matrix ($\mathbf{D}$), whose elements involve the concrete elastic modulus (E_c) and the Poisson's ratio (ν); values for these elastic parameters will be suggested later (see Appendix A for notation used).

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{e}} = \mathbf{D}^{-1} \cdot \mathrm{d}\boldsymbol{\sigma} \tag{2}$$

The plastic irreversible strains follow a non-associated flow rule (Eq. (3)), which implies that the direction of the incremental plastic strain vector ($\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}$) is normal to a plastic potential surface ($g = 0$) that differs from the loading surface ($f = 0$). It is generally accepted (e.g. Smith et al., 1989; Sfer et al., 2002) that a non-associated flow rule can describe the experimentally observed deformation capacity of concrete more closely than its associated counterpart.

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{p}} = \mathrm{d}\lambda \frac{\partial g}{\partial \sigma_{ij}} \tag{3}$$

Both failure and plastic potential surfaces are formulated in the Haigh–Westergaard stress space (Fig. 1), which is defined by the cylindrical coordinates of hydrostatic length (ξ), deviatoric length (ρ) and Lode angle (θ). These coordinates are functions of the invariants (I_1, J_2, J_3) of the principal stress tensor components ($\sigma_1 > \sigma_2 > \sigma_3$, compression negative) according to the following equations:

$$\xi = \frac{I_1}{\sqrt{3}} \quad I_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{4}$$

$$\rho = \sqrt{2J_2} \quad J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \tag{5}$$