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An inverse problem for a functionally graded elliptical plate with large deflection and slightly disturbed boundary

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Abstract

This paper deals with the inverse problem of a functionally graded material (FGM) elliptical plate with large deflection and disturbed boundary under uniform load. The properties of functionally graded material are assumed to vary continuously through the thickness of the plate, and obey a simple power law expression based on the volume fraction of the constituents. Based on the classical nonlinear von Karman plate theory, the governing equations of a thin plate with large deflection were derived. In order to solve this non-classical problem, a perturbation technique was employed on displacement terms in conjunction with Taylor series expansion of the disturbed boundary conditions. The displacements of in-plane and transverse are obtained in a non-dimensional series expansion form with respect to center deflection of the plate. The approximate solutions of displacements are solved for the first three terms, and the corresponding internal stresses can also be obtained.

Keywords: Functionally graded material; Inverse; Perturbation; Disturbed boundary; Deflection; Membrane stress; Bending stress

1. Introduction

The influence of structure elements having variable material constituents or stiffness has been an interesting problem and studied extensively in engineering applications. The purpose of varying structural properties is that it can make the structure itself become more efficient and fulfill certain special working conditions. The great advantages of being able to withstand severe temperature gradients and the ability

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to maintain structural integrity are the most important reasons for choosing FGM in structural elements, such as spacecraft and nuclear plants. For the structure problems of varying material constituents or stiffness, different methods were proposed and both closed form and approximate solutions were obtained. The inverse problem technique with properly chosen perturbation parameters seems to be one of the best methods that have been widely used for this type of problems.

In a recent paper by Huang and Shen (2004), a theoretical nonlinear vibration and dynamic response of functionally graded plates in thermal environments is presented. Approximate solutions are found by applying the Galerkin procedure and it is found that the volume fraction distribution have a significant effect on dynamic response of the plates. Ma and Wang (2003) have also presented a nonlinear bending and post-buckling of a functionally graded circular plate under mechanical and thermal loadings. Their numerical solution technique involved using the shooting method and nonlinear bending and buckling behavior is discussed in detail. Elishakoff (2001) studied the inverse buckling problem for inhomogeneous columns and derived expressions for the stiffness of the column based on the differential equation that described the buckling of the column under axial load and certain boundary conditions. With a suitable choice of parameters, it was found that the buckling load can be related to a single stiffness coefficient. Romanov et al. (2003) studied an inverse problem for a layered elastic plate where it was assumed that a point pulse force is applied at a boundary of the plate and the displacement vector is measured at the boundary. The authors found that the elasticity modules and density of layers as well as their thickness can be determined by the displacement vector in this inverse problem. Gladwell (1999) studied inverse finite element vibration problems dealing with the reconstruction of a consistent FEM model of an in-line system of 2-dof elements, fixed at one end and free at the other. The author illustrated how to construct an infinite family of such models so that each has a specified undamped frequency response at the free end, and how to construct a system with a damper at the free end so that the system has specified eigenvalues. Exact power series solutions for the axisymmetric vibrations of circular and annular membranes with continuously varying density in the general case have been presented by Willatzen (2002). In their study, a quasi-analytical approach is presented so as to find eigenfrequencies and eigensolutions in the general case where the density can be written as an infinite power series expansion in the radial co-ordinate. A general quasi-analytical model based on the Frobenius power series expansion method is described so as to handle the vibrations of circular and annular membranes with continuously varying density. Natural frequencies are finally computed for three examples of varying membrane density. Borukhov and Vabishchevich (2000) studied the numerical solution of the inverse problem of reconstructing a distributed right-hand side of a parabolic equation. They presented a numerical algorithm based on the transition method to the problem of the loaded parabolic equation. For solving this non-classical problem, they employed a special numerical method similar to the bordering method. Such a general approach can be used both for parabolic equations that are one-dimensional in space and for multidimensional ones. Hassan (2004) has presented a study involving the free transverse vibration of elliptical plates of variable thickness with half of the boundary clamped and the rest free. In this study, first four frequencies were computed by using the Rayleigh–Ritze method. Laura and Rossit (1999) studied the case of an orthotropic material and obtained an exact analytical solution for the thermal bending of clamped, anisotropic, elliptic plates. A solution for the thermomechanical deformations of an isotropic linear thermoelastic functionally graded elliptic plate rigidly clamped at the edges was obtained by Cheng and Batra (2000). A power-law type function was assigned for the through-thickness variation of the volume fraction of the ceramic phase in a metal-ceramic plate. The result of the calculations for the functionally graded plate and classical shear deformation plate theories were found to be slightly different. Muhammad and Singh (2004) used an energy method with displacement fields defined by a shape function of high order polynomials for the bending of plates with various shapes. Plates with eccentric square and circular openings were also analyzed. The transverse vibration of non-homogeneous elliptic and circular plates using the two-dimensional boundary characteristic orthogonal polynomials in the Rayleigh–Ritz