Available online at www.sciencedirect.com
ScienceDirect
International Journal of Solids and Structures 45 (2008) 378–391
INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES
www.elsevier.com/locate/ijsolstr

A 2.5-D dynamic model for a saturated porous medium: Part I. Green's function

Jian-Fei Lu [a], Dong-Sheng Jeng [b,*], Sally Williams [c]

[a] *Department of Civil Engineering, Jiangsu University, Zhenjiang, Jiangsu, 212013, PR China*
[b] *Division of Civil Engineering, School of Engineering, Physics and Mathematics, University of Dundee, Dundee DD1 4HN, Scotland, UK*
[c] *School of Civil Engineering, The University of Sydney, NSW 2006, Australia*

Received 15 July 2006; received in revised form 8 May 2007
Available online 14 September 2007

Abstract

Based on Biot's theory, the dynamic 2.5-D Green's function for a saturated porous medium is obtained using the Fourier transform and the potential decomposition methods. The 2.5-D Green's function corresponds to the solutions for the following two problems: the point force applied to the solid skeleton, and the dilatation source applied within the pore fluid. By performing the Fourier transform on the governing equations for the 3-D Green's function, the governing differential equations for the two parts of the 2.5-D Green's function are established and then solved to obtain the dynamic 2.5-D Green's function. The derived 2.5-D Green's function for saturated porous media is verified through comparison with the existing solution for 2.5-D Green's function for the elastodynamic case and the closed-form 3-D Green's function for saturated porous media. It is further demonstrated that a simple form 2-D Green's function for saturated porous media can be been obtained using the potential decomposition method.

Keywords: 2.5-D Green's function; Discrete wavenumber; Porous medium; Biot's theory; The Fourier transform

1. Introduction

Green's functions along with the boundary element method (BEM) provide a powerful tool for the study of various dynamic or static problems. For example, BEM is a useful alternative to finite element method (FEM) for the solution of problems with an infinite or semi-infinite calculation domain since FEM requires both an artificial boundary layer and discretised interior elements (Beskos, 1987, 1997). However, BEM requires only surface discretisation of the calculation domain, as the adoption of an appropriate Green's function can guarantee the satisfaction of the radiation condition at infinity automatically.

* Corresponding author. Tel.: +61293512144; fax: +61293513343.
E-mail addresses: ljfdoctor@yahoo.com (J.-F. Lu), d.jeng@civil.usyd.edu.au, dong_dundee@yahoo.co.uk (D.-S. Jeng), s.williams@civil.usyd.edu.au (S. Williams).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2007.07.025

BEM can reduce the dimensions of the problem. Nevertheless, it is often desirable to further simplify the problem to minimise the memory size and the CPU time that are required for a full 3-D BEM analysis. The 2.5-D BEM (Luco et al., 1990; Zhang and Chopra, 1991; Pedersen et al., 1994; Papageorgiou and Pei, 1998) is a technique which can be adopted when a 3-D load is applied to an infinitely long structure with a uniform cross-section. Although the structure itself can essentially be considered as 2-D, the response of the structure to loading will be 3-D. Therefore the problem can be considered as having two-and-a-half-dimensions. Various types of loading can be treated in a 2.5-D problem, including arbitrarily directed incident plane waves, point forces, distributed forces or moving loads.

Although 2.5-D BEM is well established for isotropic elastic media, the 2.5-D BEM for saturated porous media has not yet been developed, since an appropriate 2.5-D Green's function for the porous medium is not available in the literature. Conversely, the 3-D and the 2-D dynamic Green's functions for porous media are readily available in the literature (Burridge and Vargas, 1979; Norris, 1985; Bonnet, 1987; Cheng et al., 1991; Zimmerman and Stern, 1993). In general, three approaches have been adopted in the derivation of porous media Green's functions: potential decomposition method (Norris, 1985; Zimmerman and Stern, 1993); analogy with thermoelasticity (Cheng et al., 1991; Dominguez, 1991; Dominguez, 1992); and Kupradze's method (Bonnet, 1987). Although using Kupradze's method (Kupradze, 1979) the Green's function for the porous medium can be easily derived, the associated operator manipulation is often lengthy and resulting Green's functions are usually of a complicated formulation.

In this study, the 2.5-D dynamic Green's function for a saturated porous medium is derived using the potential decomposition and the Fourier transform method. Based on Biot's theory and the potential decomposition method, the governing differential equations for the two parts of the 3-D Green's function for the porous medium are obtained. Using the governing differential equations for the 3-D Green's function and the Fourier transform method, the 2.5-D Green's function for the porous medium is derived. The extreme case for the obtained 2.5-D Green's function with a vanishing porosity is compared with the 2.5-D Green's function for the single-phase elastic medium. By numerically transforming the 2.5-D Green's function back into the space domain, the new 2.5-D Green's function is compared with the existing closed-form 3-D Green's function for porous media. Finally, the 2-D Green's function for a saturated porous medium is also derived by means of a similar potential decomposition methodology and a simple formulation for the 2-D Green's function is obtained.

2. Biot's theory

The constitutive equations for a homogeneous porous medium can be expressed as (Biot, 1956a, 1962)

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}e - \alpha\delta_{ij}p \tag{1a}$$

$$p = -\alpha Me + M\zeta \tag{1b}$$

$$e = u_{i,i}, \quad \zeta = -w_{i,i} \tag{1c, d}$$

in which u_i and w_i denote the average solid displacement and the infiltration displacement of the pore fluid; ε_{ij} and e are the strain tensor and the dilatation of the solid skeleton; ζ is the volume of fluid injected into a unit volume of the bulk material; σ_{ij} is the stress of the bulk porous medium; p is the excess pore pressure and δ_{ij} is the Kronecker delta. The compressibility of the saturated porous medium is considered in terms of the solid skeleton Lamè constants, λ and μ, and Biot's parameters α and M (Biot, 1941).

The bulk density of a porous medium with solid skeleton of density ρ_s, pore fluid of density ρ_f, and porosity ϕ, can be expressed as $\rho_b = (1-\phi)\rho_s + \phi\rho_f$. Using a superimposed dot to denote the time derivative and a star (*) to denote the time convolution, the equations of motion for the bulk porous medium and the pore fluid are

$$\mu u_{i,jj} + (\lambda + \alpha^2 M + \mu)u_{j,ji} + \alpha M w_{j,ji} + F_i = \rho_b \ddot{u}_i + \rho_f \ddot{w}_i \tag{2a}$$

$$\alpha M u_{j,ji} + M w_{j,ji} + f_i = \rho_f \ddot{u}_i + m\ddot{w}_i + \frac{\eta}{k}K(t) * \dot{w}_i \tag{2b}$$

where η and k are the viscosity of the pore fluid and the permeability of the porous medium respectively; F_i and f_i are the body forces experienced by the porous medium and the pore fluid; and $m = a_\infty \rho_f/\phi$, in which a_∞ is