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Fundamental solution for transversely isotropic thermoelastic materials

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Abstract

We use the compact harmonic general solutions of transversely isotropic thermoelastic materials to construct the three-dimensional fundamental solutions for a steady point heat source in an infinite transversely isotropic thermoelastic material and a steady point heat source on the surface of a semi-infinite transversely isotropic thermoelastic material by three newly introduced harmonic functions, respectively. All components of coupled field are expressed in terms of elementary functions and are convenient to use. Numerical results for hexagonal zinc are given graphically by contours.

Keywords: Fundamental solution; Point heat source; Transversely isotropic; Thermoelastic

1. Introduction

Fundamental solutions or Green's functions play an important role in both applied and theoretical studies on the physics of solids. They are foundations of a lot of further works. For example, fundamental solutions can be used to construct many analytical solutions of practical problems when boundary conditions are imposed. They are essential in the boundary element method as well as the study of cracks, defects and inclusions.

For isotropic materials, there is well-known closed-form Kelvin fundamental solution (Banerjee and Butterfield, 1981). The first fundamental solution for transversely isotropic materials was given by Lifshitz and Rozentsveig (1947) using the Fourier transform method. Elliott (1948) gave the displacements due to a point force acting normal to the plane of isotropy. Kroner (1953) obtained explicit formulae for the displacements caused by unit force acting at the origin and parallel to the Cartesian coordinate axes. Willis (1965) obtained an expression for the fundamental solution. Sveklo (1969) solved the problem of a point force normal to the plane of isotropy using a complex method. Lejcek (1969) completed the work of Lifshitz and Rozentsveig

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(1947). Pan and Chou (1976) gave the fundamental solution in compact form. The thermal effects have not been considered in all the above works.

Sharma (1958) gave the fundamental solution of transversely isotropic thermoelastic materials in an integral form. Yu et al. (1992) gave the Green's function for a point heat source in two-phase isotropic thermoelastic materials. Recently, Chen et al. (2004) derived a compact three-dimensional general solution for transversely isotropic thermoelastic materials. In this general solution, all components of thermoelastic field are expressed by three harmonic functions.

In this paper, three-dimensional fundamental solutions for a steady point heat source in an infinite transversely isotropic thermoelastic material and a steady point heat source on the surface of a semi-infinite transversely isotropic thermoelastic material are investigated. For completeness, the general solution of Chen et al. (2004) is described in Section 2. In Sections 3 and 4, three new suitable harmonic functions are constructed for infinite and semi-infinite thermoelastic materials, respectively, in the form of elementary functions with undetermined constants by trial-and-error. The corresponding thermoelastic fields can be obtained by substituting these functions into the general solution. The undetermined constants for infinite thermoelastic material can be obtained by continuous conditions on plane $z = 0$ and the equilibrium conditions of a cylinder within $a_1 \leqslant z \leqslant a_2$ ($a_1 < 0 < a_2$) and $0 \leqslant r \leqslant b$, and the undetermined constants for semi-infinite thermoelastic material can be obtained by the boundary conditions on surface $z = 0$ and the equilibrium conditions of a cylinder within $0 \leqslant z \leqslant a$ and $0 \leqslant r \leqslant b$, where a, a_1, a_2 and b are arbitrary but should include the point source point. Numerical examples are presented in Section 5. All stress components and temperature increment are shown in the form of contours. Finally, the paper is concluded in Section 6.

2. General solutions for transversely isotropic thermoelastic materials

When the xy-plane is parallel to the plane of isotropy in Cartesian coordinates (x, y, z), the constitutive relations of transversely isotropic thermoelastic materials are

$$\begin{aligned}
\sigma_x &= c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} - \lambda_{11}\theta, \quad \tau_{yz} = c_{44}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right),\\
\sigma_y &= c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} - \lambda_{11}\theta, \quad \tau_{zx} = c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),\\
\sigma_z &= c_{13}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + c_{33}\frac{\partial w}{\partial z} - \lambda_{33}\theta, \quad \tau_{xy} = c_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),
\end{aligned} \tag{1}$$

where u, v and w are components of the mechanical displacement in x-, y- and z-directions, respectively; σ_{ij} and θ are the stress components and temperature increment, respectively; and c_{ij} and λ_{ii} are elastic and thermal modules, respectively. The relation $c_{66} = (c_{11} - c_{12})/2$ holds for materials with transverse isotropy.

In the absence of body forces, the mechanical and heat equilibrium equations are

$$\begin{aligned}
&\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0,\\
&\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0,\\
&\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0,
\end{aligned} \tag{2a}$$

$$\beta_{11}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + \beta_{33}\frac{\partial^2\theta}{\partial^2 z} = 0, \tag{2b}$$

where β_{ii} ($i = 1, 3$) are coefficients of heat conduction.

Chen et al. (2004) derived a compact general solution to Eqs. (1) and (2) as follows: