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Nonlinear dynamics of elastic rods using the Cosserat theory: Modelling and simulation

D.Q. Cao [a,*,1], Robin W. Tucker [b]

[a] School of Astronautics, Harbin Institute of Technology, P.O. Box 137, Harbin 150001, China
[b] Department of Physics, Lancaster University, Lancaster LA1 4YB, UK

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Abstract

The method of Cosserat dynamics is employed to explore the nonplanar nonlinear dynamics of elastic rods. The rod, which is assumed to undergo flexure about two principal axes, extension, shear and torsion, are described by a general geometrically exact theory. Based on the Cosserat theory, a set of governing partial differential equations of motion with arbitrary boundary conditions is formulated in terms of the displacements and angular variables, thus the dynamical analysis of elastic rods can be carried out rather simply. The case of doubly symmetric cross-section of the rod is considered and the Kirchoff constitutive relations are adopted to provide an adequate description of elastic properties in terms of a few elastic moduli. A cantilever is given as a simple example to demonstrate the use of the formulation developed. The nonlinear dynamic model with the corresponding boundary and initial conditions are numerically solved using the Femlab/Matlab software packages. The corresponding nonlinear dynamical responses of the cantilever under external harmonic excitations are presented through numerical simulations.

Keywords: Cosserat model; Nonlinear dynamics; Slender rod; Modelling and simulation; Femlab; Matlab

1. Introduction

Nonlinear dynamic analysis of slender elastic structures under external forces and torques and parametric excitations remains an active area of study. Such a study can find application in accelerating missiles and space crafts, components of high-speed machinery, manipulator arm, microelectronic mechanical structures (MEMS), components of bridges (such as towers and cables) and other structural elements. Considerable attention has been devoted to the study of nonlinear dynamics of rods or beams subject to both external and parametric excitations (see, for example, Nayfeh and D.T.Mook, 1979; Saito and Koizumi, 1982; Kar and Dwivedy, 1999 and the references cited therein). While attention so far has mainly been devoted to the

* Corresponding author. Tel.: +86 (0) 451 86414479; fax: +86 (0) 451 86402822.
E-mail addresses: dqcao@hit.edu.cn (D.Q. Cao), r.tucker@lancaster.ac.uk (R.W. Tucker).
1 On leave from the Department of Physics, Lancaster University, Lancaster LA1 4YB, UK.

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study of planar, nonlinear dynamic analysis of beams, research has been done concerning nonplanar, nonlinear motions of beams. Bolotin (1964) addressed such motions in beams, but restricted himself to consideration of nonlinear inertia terms and stability of the planar response. Crespo da Silva and Glynn (1978a) formulated the equations of motion describing the nonplanar, nonlinear dynamics of an inextensional beam. The nonplanar, nonlinear forced oscillations of a cantilever are then analyzed in Crespo da Silva and Glynn (1978b) using the perturbation method. Cartmell (1990) and Forehand and Cartmell (2001) derived the nonlinear equation of motion for the in-plane and out-of-plane forced vibration of cantilever beams with a lumped mass.

The studies mentioned above are restricted to systems with no extension of the beam's neutral axis and no warping or shear deformation. Crespo da Silva (1988a,b) investigated the problem of nonlinear dynamics of the nonplanar flexural-torsional-extensional beams, but the effects of rotary inertia and shear deformation were neglected. Thus, cross-sectional dimensions of the beam were assumed to be small enough in comparison to the beam length. However, the shear deformation may be of considerable importance and can not be negligible for studying the vibration of high frequencies when a comparative short rod is investigated. In such a case, the effect of shear deformation should be taken into account for. For the planar problem, although such effects can be included by using the Timoshenko beam theory, most of the studies are limited to the determination of natural frequencies and eigenfunctions (Huang, 1961; Bishop and Price, 1977; Grant, 1978; Bruch and Mitchell, 1987). For the three-dimensional problem, we refer to Simo and Vu-Quoc (1988, 1991) for a geometrically exact rod model incorporating shear and torsion-warping deformation. Further studies on the dynamic formulation of sandwich beams have been presented in Vu-Quoc and Deng (1995) and Vu-Quoc and Ebcioglu (1995) based on the geometrically-exact description of the kinematics of deformation. Moreover, Esmailzadeh and Jalili (1998) investigated the parametric response of cantilever Timoshenko beams with lumped mass, but restricted themselves to consideration of nonlinear inertia terms. Based on the geometrically-exact model of sliding beams, parametric resonance has been presented in Vu-Quoc and Li (1995), where the beams can undergo large deformation with shear deformation accounted for.

In the case involving the full dynamic response, the strong nonlinearity and fully coupling introduce a challenge for solving the partial differential equations of motion of rods. We refer to Rubin (2001) for a formulation of a numerical solution procedure for three-dimensional dynamic analysis of rods by modelling the rod as a set of connected Cosserat points, also Rubin and Tufekci (2005) and Rubin (2000) for three-dimensional dynamics of a circular arch and shells using the theory of a Cosserat point, respectively. A Galerkin projection has been applied to discretize the governing partial differential equations of sliding beams in Vu-Quoc and Li (1995) and sandwich beams in Vu-Quoc and Deng (1997). Recently, a new modelling strategy has been proposed in Cao et al. (2006) to discretize the rod and to derive the ordinary differential equations of motion with third order nonlinear generic nodal displacements. This modelling strategy has been successfully used to investigate the nonlinear dynamics of typical MEMS device that comprises a resonator mass supported by four flexible beams (Cao et al., 2005).

In this paper we explore the nonplanar, nonlinear dynamics of an extensional shearable rod by using the simple Cosserat model. The method of Cosserat dynamics for elastic structures is employed since it can accommodate to a good approximation the nonlinear behavior of complex elastic structures composed of materials with different constitutive properties, variable geometry and damping characteristics (Antman, 1991; Antman et al., 1998; Tucker and Wang, 1999; Cull et al., 2000; Gratus and Tucker, 2003). With arbitrary boundary conditions, the Cosserat theory is used to formulate a set of governing partial differential equations of motion in terms of the displacements and angular variables, describing the nonplanar, nonlinear dynamics of an extensional rod. Bending about two principal axes, extension, shear and torsion are considered, and care is taken into account for all the nonlinear terms in the resulting equations. As an example, a simple cantilever is given to demonstrate the use of the formulation developed. The nonlinear dynamic model with the corresponding boundary and initial conditions are numerically solved using the commercially available software packages Femlab and Matlab. Corresponding nonlinear dynamical responses of the cantilever under external harmonic excitations are presented and discussed through numerical simulations.

The following conventions and nomenclature will be used through out this paper. Vectors, which are elements of Euclidean 3-space $\mathcal{R}^3$, are denoted by lowercase, bold-face symbols, e.g., $\mathbf{u}$, $\mathbf{v}$; vector-valued functions are denoted by lowercase, italic, bold-face symbols, e.g., $\boldsymbol{u}$, $\boldsymbol{v}$; tensors are denoted by upper-case, bold-face symbols, e.g., $\mathbf{I}$, $\mathbf{J}$; matrices are denoted by upper-case, italic, bold-face symbols, e.g., $\boldsymbol{M}$, $\boldsymbol{K}$. The symbols