Available online at www.sciencedirect.com
ScienceDirect
International Journal of Solids and Structures 45 (2008) 540–550
INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES
www.elsevier.com/locate/ijsolstr

Stresses in accreted planetary bodies

Jon Kadish [a], J.R. Barber [a,b,*], P.D. Washabaugh [c], D.J. Scheeres [c]

[a] *Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109-2125, USA*
[b] *Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI 48109-2125, USA*
[c] *Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109-2140, USA*

Received 21 March 2007; received in revised form 9 July 2007
Available online 26 August 2007

Abstract

An explicit analytical solution is obtained for the stress field in an accreted triaxial ellipsoid under the influence of self-gravitation and rotation. Material is assumed to attach to the surface of the accreting body in a stress-free state, after which it behaves elastically. The results differ significantly from the classical elasticity solutions that are based on the assumption that the body is fully formed before the loading is applied. These results are relevant to the strengths of accreted planetary bodies such as comets and asteroids.

The solution allows both the magnitude and direction of the angular velocity to be a general function of the time-like parameter defining the progress of accretion. Simple closed-form expressions are given for two special cases—the ellipsoid accreting at constant angular velocity and the sphere accreting with an angular velocity vector that precesses through 90° during the accretion process. A Mathematica notebook permitting the solution of other problems can be downloaded from the website http://www-personal.umich.edu/jbarber/ellipsoid.nb.

Keywords: Accretion; Elasticity; Spherical harmonics; Residual stresses; Asteroids; Comets; Planets; Rotational stresses

1. Introduction

The stress field in an object depends on the manner in which it was constructed. For example, the stresses in a body that has solidified from a melt are influenced by its original liquid state in which the stresses are hydrostatic (Pedroso and Domoto, 1973); calculation of the stresses in dam embankments must account for the fact that these structures are constructed by placing pre-formed blocks one on top of the other (Clough and Woodward, 1967); stresses in biological tissues are influenced by growth emanating from the material's bulk (e.g., Rodriguez et al., 1994); and stresses in spherulites are influenced by transformational strains during the growth process (Dryden, 1987; Burns, 1996). In this paper, we shall investigate how the growth process affects the stress fields of small planetary bodies.

* Corresponding author. Address: Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109-2125, USA. Tel.: +1 7349360406; fax: +1 7346156647.
E-mail address: jbarber@umich.edu (J.R. Barber).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2007.08.008

4.6 Billion years ago, all the mass currently contained within the planets, moons, asteroids, and comets of the solar system was in the form of dust and gas that orbited the Sun in a large cloud called the solar nebula. The growth of kilometer sized objects from sub-micron sized dust grains occurred by the collisional and gravitational evolution of a swarm of particles. Growth in this manner, or growth by the continual deposition of material onto an object's surface, is known as a process of accretion (e.g., Weidenschilling, 2000). The only forays into calculating the stress fields of accreted planetary bodies were made by Brown and Goodman (1963) and Kadish et al. (2005). Brown and Goodman (1963) found the stress field of an accreted sphere under the influence of self-gravitation, and made brief mention of the relationship between their results and the stress field of the Earth. Kadish et al. (2005) extended Brown and Goodman's results to include the effect of loading due to rotation, and applied the results to investigate possible disruption mechanisms of small planetary bodies. They found that the stress field is significantly influenced by the history of the rotational speed during the accretion process.

Of course, not all objects of the solar system are spheres and a better approximation is to calculate an ellipsoidal fit to these objects. In this paper, we extend the solution of Brown and Goodman (1963) and Kadish et al. (2005) to that of an accreted ellipsoid.

2. Stress field in an accreted body

We consider the accretion of a triaxial ellipsoid whose shape remains constant while its size grows. The semi-axes of the final ellipsoid after growth is completed are denoted by a_i, $i = 1, 2, 3$ and we define a dimensionless effective radius r as

$$r \equiv \sqrt{\frac{x_i^2}{a_i^2}}, \tag{1}$$

where x_i are coordinates of a body-fixed Cartesian coordinate system aligned with the semi-axes of the ellipsoid and the summation convention is implied. The instantaneous boundary of the body at time t is then defined by $r = s(t)$ where s is called the growth parameter.

The ellipsoid grows from $s = 0$ to $s = 1$ due to the accretion of particles in an orbiting dust cloud. These particles add mass and angular momentum to the accreted body, and hence may change its angular velocity vector $\mathbf{\Omega}(s)$, which therefore becomes a function of the growth parameter s. Dones and Tremaine (1993) discuss the form that this function may take, based on the accumulation of mass and angular momentum of an accreting body in a cloud of particles. The accretion process is extremely slow—accretion of a typical asteroid may take 10^6 years—so it is reasonable to neglect the inertia forces associated with angular acceleration due to accretion.

If an ellipsoidal body rotates about a non-principal axis and experiences no external forces, the inertia forces associated with centripetal acceleration have a non-zero torque resultant about a perpendicular axis, causing the axis of rotation to precess or 'nutate'. In a frame of reference fixed in the body, the instantaneous axis of rotation traces out a cone centered on the major principal axis. The resulting body forces and the oscillating elastic stress field that they generate were given by Sharma et al. (2005), who used their results to estimate the rate at which the nutation would become damped out, causing the axis of rotation to approach the major principal axis.

In the present paper, we shall restrict attention to cases where the accelerations due to the time derivative $\dot{\mathbf{\Omega}}$ of the angular velocity vector can be neglected in comparison with the centripetal accelerations. This will be a reasonable approximation if the nutation period is long relative to that of diurnal rotation, which requires either that the ellipticity of the body be not too large, or that the axis of rotation be at (in which case there is no nutation) or near to a principal axis.[1]

[1] It is perhaps worth noting that if these conditions are not met and there is rapid nutation and associated periodically varying accelerations, the effect of the accretion process would be to develop a spatially periodic residual stress field. However, because the accretion process is so slow, the spatial wavelength of this field would be relatively short and may in fact be smaller than the size of the typical accreted particle.