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Analytical and numerical solution for a elastic pipe bend at in-plane bending with consideration for the end effect

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Abstract

The authors proposed an analytical method for the analysis of the end effect in a pipe bend loaded by a bending moment with consideration for the action of internal pressure. The method is based on the use of simplifying hypotheses and is reduced to the solution of a system of fourth-order differential equations along the axial coordinate with respect to unknown coefficients in the expansion for tangential displacements. An approximate analytical solution, which has a trapezoidal structure and is written in terms of Krylov's functions, has been obtained. Boundary conditions are formulated in terms of the tangential and longitudinal displacements and axial and shearing stress resultant. For the flexibility factor, analytical solutions are presented in the case where a bend is approximated by a rigid restraint on both ends. To verify the analytical solution and its applicability limits, two numerical procedures were developed, which are based on the finite difference method and the reduction to the Kochi problem by the expansion of the unknowns in the Fourier series over the circumferential coordinate. The authors compare the results obtained with data from the literature, discuss the advantages and disadvantages of the methods, and present recommendations for their practical application.

Keywords: Pipe bend; Shell; End effect; Analytical solution; Numerical procedure; Flexibility factor

1. Introduction

A pipe bend is a critical component of piping systems used in power, petrochemical, and other industries. Calculation of its stress state is an important and complex problem. A peculiar feature of a deformation of a bend is that additional transverse forces resulting in ovalization of the cross-section occur when external bending moments are applied perpendicular to the bend axis. The ovalization of the cross-section leads to an increase in the bend flexibility factor K as compared with a straight pipe of the same cross-section and affects considerably the stress state.

There exist two dimensionless parameters predetermining the difference between a pipe bend and a straight pipe. The first is the parameter of curvature α representing the ratio of the bend cross-section radius R to the

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bend axis radius A, i.e., $\alpha = R/B$, the second is the parameter of flexibility λ determined from the following relationship: $\lambda = \frac{R^2}{Bt}$, where t is the pipe wall thickness. The larger these parameters, the stronger the differences in the flexibilities and stress distributions in a straight pipe and a pipe bend. A pipe bend is usually considered as a toroidal shell and is solved by appropriate methods. It should be noted that in the literature on the shell theory, many of theoretical methods and practical examples of calculations are associated with the solutions for a pipe bend. This problem attracts investigators because of a great variety of effects and tasks encountered in the analysis of toroidal shells. By convention, all these tasks can be divided into two categories.

The first category comprises the so-called far end (or local) solutions, i.e., the solutions for very long pipe bend wherein the distribution of internal forces in every bend cross-section is proportional to the values of the external global bending moments. A correct solution to this problem for in-plane bending in the case of small α and λ was first proposed by Karman (1911) although the main features of the bend deformation had been described by Dubyaga (1909) somewhat earlier. Later many publications generalized Karman's results by taking into account large values of λ (Beskin, 1945; Clark and Reissner, 1951) and α (Cross, 1952). It was found both experimentally and theoretically that, in the case of out-of-plane bending (orthogonal bending), ovalization also occurs and the bend flexibility increases similarly to in-plane bending (normal bending) (Vigness, 1943). The effect of the internal pressure manifests itself in the increase in the rigidity of the cross-section, and its analysis requires consideration of a geometrically nonlinear problem of deformation. It was considered for a normal bending in Kafka and Dunn (1956), and a more general analysis of the spatial bending is made in (Rodabaugh and George, 1957).

This category of the problems was studied well enough, and specific analytical expressions were derived for the stress distribution and flexibility factor, which have been included in the existing piping calculation standards. As an example, the so-called simplified analysis (NB 3600) included in part III of ASME Boiler and Pressure Code (1995) should be mentioned wherein the formulas proposed in (Rodabaugh and George, 1957) have been used until now.

For the most part, the above methods involve the traditional approach proposed by Karman and are based on the principle of minimization of the elastic energy functional. The present authors endorse approaches based on the exact solution of differential equilibrium equations and geometrical equations. The method of solution proposed by the authors (Orynyak and Radchenko, 2004a,b,c) is notable in that it not only predicts the ovalization stresses in a pipe bend as a shell, but it also yields general equations for the deformation of the shell axis as a line of the cross-section centers of mass. Thus, a pipe bend can be considered as one element (a curvilinear rod) of a complex piping system whose stress-strain state is calculated using the theory of rods.

Name the above-mentioned solutions as the far end (local) ones. They provide the general understanding of the pipe bend behavior for a long enough bend and it is the starting point in treating the end effect. Thus the second category of the tasks relates to end effects occurring near the junction of a pipe bend with other elements of the piping system.

Beginning in the 1920s, many works were dedicated to the experimental investigation of this problem (Axelrad and Ilyin, 1972); they were particularly intensive in view of the development of new ASME norms for pipeline stress analysis early in the 50s. The most careful studies were performed by Pardue and Vigness (1951). Based on their results, Markl (1955) formulated general recommendations on the practical consideration of the influence of the fitting and fastening conditions on the pipe bend stresses and flexibility, which were included in various international standards. Meanwhile, as early as in 1972 Axelrad and Ilyin (1972) pointed out the shortcomings of those recommendations. Thus, in particular, the length of the transition zone must depend on the absolute length of the pipe bend rather than on its angular length.

Since end effects often reduce the actual stresses compared to those calculated according to far end solution their consideration is a strong stimulus for reducing the consumption of the piping material. In the technical literature there is a comprehension of the shortcomings associated with the disregard for the end effects or their consideration on the basis of Markl's recommendations. For this reason, in the ASME Boiler and Pressure Code (1995), an additional provision has been made for a so-called detailed analysis (NB 3200) performed generally with the use of the finite element method (FEM).

The first works on the pipe bend calculation using the FEM appeared in the 70s, and as an example, we mention works (Natarajan and Blomfield, 1975; Ohtsubo and Watanabe, 1977) wherein a numerical analysis of the influence of the end conditions on the flexibility and stress distribution was made. In the early 1980s,