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ScienceDirect
International Journal of Solids and Structures 44 (2007) 1550–1562
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

State-space approach of two-temperature generalized thermoelasticity of one-dimensional problem

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Received 14 November 2005; received in revised form 19 June 2006
Available online 27 June 2006

Abstract

In this paper, we will consider a half-space filled with an elastic material, which has constant elastic parameters. The governing equations are taken in the context of the two-temperature generalized thermoelasticity theory [Youssef, H., 2005a. The dependence of the modulus of elasticity and the thermal conductivity on the reference temperature in generalized thermoelasticity for an infinite material with a spherical cavity, J. Appl. Math. Mech., 26(4), 4827; Youssef, H., 2005b. Theory of two-temperature generalized thermoelasticity, IMA J. Appl. Math., 1–8]. The medium is assumed initially quiescent. Laplace transform and state space techniques are used to obtain the general solution for any set of boundary conditions. The general solution obtained is applied to a specific problem of a half-space subjected to thermal shock and traction free. The inverse Laplace transforms are computed numerically using a method based on Fourier expansion techniques. Some comparisons have been shown in figures to estimate the effect of the two-temperature parameter.

Keywords: Thermoelasticity; Generalized thermoelasticity; Two-temperature; State space

1. Introduction

Because of the advancement of pulsed lasers, fast burst nuclear reactors and particle accelerators, etc., which can supply heat pulses with a very fast time-rise Bargmann (1974), Anisimov et al. (1974), Boley (1980), Qiu and Tien (1993), Tzou (1997), Chen et al. (2004), Naotak et al. (2003); generalized thermoelasticity theory is receiving serious attention of different researchers. The development of the second sound effect has been reviewed by Chandrasekhariah (1986). Now, mainly two different models of generalized thermoelasticity are being extensively used-one proposed by Lord and Shulman (1967) and the other proposed by Green and Lindsay (1972). The L–S theory suggests one relaxation time and according to this theory, only Fourier's heat conduction equation is modified; while G–L theory suggests two relaxation times and both the energy equation and the equation of motion are modified. A method for solving coupled thermoelastic problems by using

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2006.06.035

Nomenclature

λ, μ	Lame's constants
ρ	density
C_E	specific heat at constant strain
t	time
T	temperature
T_0	reference temperature
α_T	coefficient of linear thermal expansion
γ	$=\alpha_T(3\lambda + 2\mu)$
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
K	thermal conductivity
τ_0	relaxation times
c_0	$=\sqrt{\frac{\lambda+2\mu}{\rho}}$ longitudinal wave speed
η	$=\frac{\rho C_E}{K}$ the thermal viscosity
ε	$=\frac{\gamma}{\rho c_E}$ dimensionless thermoelastic coupling constant
a	$a > 0$ two-temperature parameter
β	$= a c_0^2 \eta^2$ dimensionless two-temperature parameter
α	$=\frac{\gamma T_0}{\lambda+2\mu}$ dimensionless mechanical coupling constant

the state-space approach was developed by Bahar and Hetnarski (1977a,b, 1978). Erbay and Suhubi (1986) studied longitudinal wave propagation in an infinite circular cylinder, which is assumed to be made of the generalized thermoelastic material, and thereby obtained the dispersion relation when the surface temperature of the cylinder was kept constant. Generalized thermoelasticity problems for an infinite body with a circular cylindrical hole and for an infinite solid cylinder were solved respectively by Furukawa et al. (1990). A problem of generalized thermoelasticity was solved by Sherief (1993) by adopting the state-space approach. Chandrasekaraiah and Murthy (1993) studied thermoelastic interactions in an isotropic homogeneous unbounded linear thermoelastic body with a spherical cavity, in which the field equations were taken in unified forms covering the coupled, L–S and G–L models of thermoelasticity. The effects of mechanical and thermal relaxations in a heated viscoelastic medium containing a cylindrical hole were studied by Misra et al. (1987). Investigations concerning interactions between magnetic and thermal fields in deformable bodies were carried out by Maugin (1988) as well as by Eringen and Maugin (1989). Subsequently Abd-Alla and Maugin (1990) conducted a generalized theoretical study by considering the mechanical, thermal and magnetic field in centrosymmetric magnetizable elastic solids.

Within the theoretical contributions to the subject are the proofs of uniqueness theorems under different conditions by Ignaczak (1979, 1982) and by Sherief (1987). The state space formulation for problems not containing heat sources was done by Anwar and Sherief (1988a) and the boundary element formulation was done by Anwar and Sherief (1988b). Some concrete problems have also been solved. The fundamental solutions for the spherically symmetric spaces were obtained by Sherief (1986). Sherief and Anwar (1986, 1994) have solved some two dimensional problems, while Sherief and Hamza (1994) have solved some two dimensional problems and studied the wave propagation in this theory. El-Maghraby and Youssef (2004) used the state space approach to solve a thermomechanical shock problem. Sherief and Youssef (2004) get the short time solution for a problem in magnetothermoelasticity. Youssef (2005) constructed a model of the dependence of the modulus of elasticity and the thermal conductivity on the reference temperature and solved a problem of an infinite material with a spherical cavity.

Chen and Gurtin (1968), Chen et al. (1968, 1969) have formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperatures, the conductive temperature ϕ and the thermodynamic temperature T. For time independent situations, the difference between these two temperatures is