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A domain-independent integral for computation of stress intensity factors along three-dimensional crack fronts and edges by BEM

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Abstract

The present work deals with an evaluation of stress intensity factors (SIFs) along straight crack fronts and edges in three-dimensional isotropic elastic solids. A new numerical approach is developed for extraction, from a solution obtained by the boundary element method (BEM), of those SIFs, which are relevant for a failure assessment of mechanical components. In particular, the generalized SIFs associated to eigensolutions characterized by unbounded stresses at a neighbourhood of the crack front or a reentrant edge and also that associated to T-stress at the crack front can be extracted. The method introduced is based on a conservation integral, called H-integral, which leads to a new domain-independent integral represented by a scalar product of the SIF times some element shape function defined along the crack front or edge. For sufficiently small element lengths these weighted averages of SIFs give reasonable pointwise estimation of the SIFs. A proof of the domain integral independency, based on the bi-orthogonality of the classical two-dimensional eigensolutions associated to a corner problem, is presented. Numerical solutions of two three-dimensional problems, a crack problem and a reentrant edge problem, are presented, the accuracy and convergence of the new approach for SIF extraction being analysed.

Keywords: Stress intensity factors; Three-dimensional cracks; Edge singularities; Vertex singularities; Conservation integral; H-integral; Domain-independent integral; Boundary element method

1. Introduction

The evaluation of stress intensity factors (SIFs) associated to cracks has been a major task in linear elastic fracture mechanics since Williams (1957) and Irwin (1957) demonstrated that these parameters control the singular stress field near the crack tip. It has been shown experimentally that there is a critical value of SIF which

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controls fracture initiation. To assure the structural integrity of mechanical components in industry the SIF concept is widely used.

Analogous parameters which govern the singular stress field at the neighbourhood of an edge, corner in two-dimensions (Wieghardt, 1907; Williams, 1952), or a three-dimensional polyhedral vertex (Benthem, 1977; Bažant and Estenssoro, 1979), are usually also referred to as (generalized) SIFs. While the role of the SIF for cracks is well understood, the role of the SIFs for edges (corners in 2-D) in failure is under study currently. Most of the failure criteria at reentrant edges in brittle elastic materials are based on the SIF concept, see for example Seweryn (1994), Dunn et al. (2001) and Yosibash et al. (2004).

There are several numerical methods for SIFs calculation in the finite element and boundary element methods (FEM and BEM). Most of them could be classified in local or global methods, the former dealing with the local variables defined near the crack tip whereas the latter are based on the far field variables or variables associated to the whole domain.

The local methods, like the quarter-point singular element (Henshell and Shaw, 1975; Barsoum, 1976) and displacement and stress extrapolation methods (Chan et al., 1970), may be very sensitive, their accuracy being strongly dependent on the refinement near the crack front or edge in the discretized model.

The global methods are based, for example, on conservation integrals, like *J*-, *M*-, *L*- and *H*-integrals. Conservation integrals constitute a more robust approach since they eliminate the need to solve local singular stress fields very accurately. The *J*-integral method is an energy approach proposed originally for plane problems (Eshelby, 1956; Rice, 1968) and appears to be the first path-independent integral proposed to evaluate SIFs. To extract SIFs in mixed-mode cracks using *J*-integral, it is necessary to decouple the stress and strain fields into the symmetric mode I and both antisymmetric modes II and III, see Huber et al. (1993) and Rigby and Aliabadi (1993). The interaction integral or two-state conservation integral, based on the superposition of the two equilibrium states, was successfully used, for example, by Miyazaki et al. (1993) and Cisilino and Ortiz (2005), respectively, for 2-D and 3-D interface cracks using BEM. The *M*-integral is an attractive methodology formulated originally by Knowles and Sternberg (1978). Two-state conservation laws obtained from the *M*-integral (Chen and Shield, 1977) have recently been adapted to the evaluation of SIFs at 2-D corners and 3-D edges, respectively, by Im and Kim (2000) and Lee and Im (2003) using FEM. The two-state *L*-integral was employed by Choi and Earmme (1992) to compute SIFs for circular arc-shape cracks. The so-called *H*-integral is in fact the Bueckner (1973) work-conjugate integral, which is derived from the well-known 2nd Betti reciprocal theorem. It has been applied in several works for SIF calculation at 2-D corners in isotropic, anisotropic and dissimilar materials, see, for example, Sinclair et al. (1984), Carpenter (1984, 1995) and Szabó and Babuška (1991), for applications to homogeneous isotropic corners. For 3-D crack problems a path-independent expression was derived by Meda et al. (1998) starting from the *H*-integral. All of these methodologies based on path-independent integrals necessarily require the use of an auxiliary field. The fact that the *H*-integral computation requires only evaluation of natural variables like displacements and stresses represents an inherent advantage of this approach, which can imply its better accuracy in comparison with other conservation integrals that require gradients of displacements and/or stresses.

The BEM is ideally suited for the evaluation of conservation integrals. This is because the required displacements and stresses (and also their derivatives if required) at internal points can be obtained in BEM with a high accuracy from their boundary integral representations, as opposed to other numerical techniques, like FEM. For 3-D calculations, the conservation integrals are typically used in their so-called surface- or domain-independent form. When the surface-independent form of conservation integrals is adopted, the integration is performed along a contour in a plane perpendicular to the crack front and also over the surface enclosed by this contour, see, for example, Huber et al. (1993), Rigby and Aliabadi (1993) for *J*-integral applications and Meda et al. (1998) for *H*-integral applications. Li et al. (1985) and Shih et al. (1986) using FEM, and more recently Ortiz and Cisilino (2005) using BEM, showed that *J*-integral in its domain-independent expression is versatile, efficient and simple to implement numerically. To develop the domain-independent *J*-integral, an auxiliary function q, which can be interpreted as a virtual crack front advance, is incorporated, Li et al. (1985) and Shih et al. (1986).

The present work has been motivated by the need to develop a numerical tool to compute in a direct and precise way the SIFs along 3-D crack fronts and edges. For this purpose a new 3-D *H*-integral formulation will be developed starting from its 2-D formulation. The domain-independent representation for this integral is