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# An asymptotically correct classical model for smart beams

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**Abstract**

An asymptotically correct classical beam model has been developed for smart slender structures using the variational asymptotic method. Taking advantage of the slenderness of the structure, we asymptotically split the original three-dimensional electromechanical problem into a two-dimensional electromechanical cross-sectional analysis and a one-dimensional beam analysis. The one-dimensional beam analysis could be geometrically nonlinear or linear depending whether the original three-dimensional analysis is geometrically nonlinear or linear. The cross-sectional analysis, implemented using the finite element method, provides an asymptotically correct, one-dimensional constitutive model for smart slender structures without a priori assumptions regarding the geometry of the cross section, the distribution of the electric field, and the location of smart materials, such as embedded or surface mounted. Several examples are used to validate the accuracy of the present theory with available results in the literature and three-dimensional commercial finite element packages.


*Keywords:* Variational asymptotic method; Smart beams; VABS

## 1. Introduction

In recent years, there has been a strong interest in applying active materials to make structures ‘‘smart’’ (Chopra, 2002; Chee et al., 1998; Saravanos and Heyliger, 1999). Active materials such as piezoelectrics are capable of sensing and reacting to external stimuli and thus provide a new dimension for us to improve the performance of modern and future structural systems (Noor et al., 2000). Despite tremendous advances in the technology of smart-structures, the analytical predictive capabilities for smart structures are still very limited in comparison to those for conventional composite structures. Many engineering structures can be analyzed using beam models if one dimension is much larger than the other two dimensions of the structure. For this very reason, such structures are usually termed as smart beams. To take advantage of this geometrical feature, different researchers have proposed various smart beam models to capture the behavior associated with the two small dimensions eliminated in the final one-dimensional (1D) beam analysis.

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Generally speaking, most of the studies in the literature can be classified as engineering models which are based on a priori kinematic assumptions, asymptotic models which are derived based on asymptotic expansions of the three-dimensional (3D) quantities, and models based on the Saint–Venant principle.

Engineering models begin with assuming some kind of distribution through the cross section for the 3D quantities in terms of the 1D quantities defined on a chosen beam axis. These models dominate the literature on the modeling of smart beams. They can be further classified as uncoupled models (Crawley et al., 1990; Robbins and Reddy, 1991; Park et al., 1996; Zhang and Sun, 1996; Smyser and Chandrashekhara, 1997) if only actuation of smart materials is treated, or coupled models (Saravanos and Heyliger, 1995; Raja et al., 2000) if both actuation and sensing capabilities of smart materials are treated simultaneously in the modeling process. These models use assumptions mainly based on engineering intuition and have clear physical meaning. The numerical implementation of such models can be developed straightforwardly from a variational statement. However, most of the a priori kinematic assumptions are natural extensions from those for homogeneous, isotropic beams and cannot be easily justified for heterogeneous structures made with anisotropic materials such as smart beams. Moreover, there is no rational way for the analysts to determine the loss of accuracy and what kind of assumptions (i.e., single-layer versus layerwise, first-order versus higher-order) should be used for sufficient accuracy while keeping a reasonable computational cost.

Unlike engineering models, asymptotic models reduce the original 3D problem into a sequence of 1D beam models by taking advantage of the small parameters inherent in the structure (Altay and Dokmeci, 2003). The conventional practice is to apply a formal asymptotic expansion directly to the system of governing differential equations of the 3D problem and successively solve the 1D field equations from the leading order to higher orders. Although these models are mathematically elegant and rigorous, sometime it is difficult to interpret the physical meanings of certain terms at a particular order level, and it is very difficult, to implement these theories numerically. These methods become intractable for a complex problem such as smart beams. Although there are some conventional asymptotic models for smart plates developed (Reddy and Cheng, 2001), such models for smart beams are rarely developed.

The variational asymptotic method (VAM) introduced by Berdichevsky (1979) can be used to construct beam models with both merits of engineering models (viz., systematic and easy numerical implementation) and asymptotic models (viz., without a priori kinematic assumptions) (Yu et al., 2002). Recently, Cesnik et al. applied VAM to model smart beams with active twist using active fiber composites. They have developed classical models for smart thin-walled beams (Cesnik et al., 2001), smart solid beams (Cesnik and Ortega-Morales, 2001), and refined models for smart solid beams (Cesnik and Palacios, 2003; Palacios and Cesnik, 2005). All these models have been implemented in the computer code UM/VABS. Using the temperature analogy, the effects of smart materials were initially modeled by assuming a constant and known electric field inside the active material (Cesnik et al., 2001; Cesnik and Ortega-Morales, 2001; Cesnik and Palacios, 2003). Later in Palacios and Cesnik (2005), a coupled analysis has been carried out using a modal procedure that allows arbitrary definition of 1D elastic and electric variables. In this work, the prescribed potential in active material is expressed through a set of independent (and given) electric modes and their dimensionless amplitudes. The induced potential in active material is obtained from a constrained minimization problem over the cross section.

There are another significant body of literature on beam models based on the celebrated Saint–Venant principle (Giavotto et al., 1983; Dong et al., 2001). The 3D displacements are represented using a Ritz-type approximation in terms of six rigid body motions (three translations and three rotations) of the cross section, which are only functions of the beam axis, and warping functions which strain the cross section. Then variational principles such as the principle of virtual work or the principle of minimum total potential energy can be used to derive a set of ordinary differential equations in terms of the beam axis and the coefficients are unknown functions of the two cross-sectional coordinates. The advantage of this method is that both "central" solution (the beam problem) and "extremity" solution (end effects) can be analyzed within the same framework, although it is not trivial to extend this method to geometrically nonlinear analysis. This method has also been generalized to deal with beams made of smart materials (Ghiringhelli et al., 1997; Taciroglu et al., 2004).

In this paper, we start from the original, geometrically nonlinear, 3D formulation of the active structure and rigorously decouple it into a two-dimensional (2D) coupled cross-sectional analysis and a 1D geometrically nonlinear beam analysis. No assumptions have been made on the distribution of mechanical and