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Thermoelasticity of bodies with microstructure and microtemperatures

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Abstract

This paper is concerned with a linear theory of thermodynamics for elastic materials with microstructure, whose microelements possess microtemperatures. It is shown that there exists the coupling of microrotation vector field with the microtemperatures even for isotropic bodies. Uniqueness and continuous dependence results are presented. The theory is used to establish the solution corresponding to a concentrated heat source acting in an unbounded continuum.

Keywords: Elastic bodies with microstructure; Thermoelasticity with microtemperatures; Continuous dependence results; Microstretch continua; Concentrated loads

1. Introduction

The origin of the modern theories of a continuum with microstructure goes back to papers by Ericksen and Truesdell (1958), Mindlin (1964), Eringen and Suhubi (1964) and Green and Rivlin (1964). Green (1965) has established the connection of the theory of multipolar continuum mechanics and the other theories. Much of the theoretical progress in the field is discussed in the books of Kunin (1983), Ciarletta and Ieşan (1993) and Eringen (1999). In the theory of micromorphic bodies formulated by Eringen and Suhubi (1964, 1999) the material particle is endowed with three deformable directors and the theory introduces nine extra degrees of freedom over the classical theory. On the basis of the theory of bodies with inner structure, Grot (1969) has established a theory of thermodynamics of elastic bodies with microstructure whose microelements possess microtemperatures. The Clausius–Duhem inequality is modified to include microtemperatures, and the first-order moment of the energy equations are added to the usual balance laws of a continuum with microstructure. The theory of micromorphic fluids with microtemperatures has been studied in various papers (see, e.g., Koh, 1973; Riha, 1975, 1977; Verma et al., 1979). Riha (1976) has presented a study of heat conduction in materials with microtemperatures. Experimental data for the silicone rubber containing spherical aluminium particles and for human blood were found to conform closely to predicted theoretical thermal conductivity.

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A theory of thermoelasticity with microtemperatures, in which the microelements can stretch and contract independently of their translations has been studied by Ieşan (2001). This is the simplest thermomechanical theory of elastic bodies that takes into account the microtemperatures and the inner structure of the materials. The theory introduces one mechanical extra degree of freedom over the classical theory. The theory of thermoelasticity with microtemperatures has been investigated by various authors. Casas and Quintanilla (2005) have studied the problem of stability. The theory of steady vibrations has been investigated by Scalia and Svanadze (2006) and Svanadze (2003, 2004).

Eringen (1999) has defined a class of micromorphic solids called microstretch solids. The material particles of these materials have seven degrees of freedom: three displacements, three microrotations and one microstretch. The microstretch continuum can model various porous media filled with gas or inviscid fluids, composite materials reinforced with chopped elastic fibers, mixtures with breathing elements and biological fluids.

In the present paper we use the results established by Grot (1969) to derive a linear theory of microstretch elastic solids with microtemperatures. This theory introduces three extra degrees of freedom over the theory presented by Ieşan (2001). A material particle is then equipped with the degrees of freedom for rigid rotations, in addition to the classical translation degrees of freedom and the microstretch. An interesting aspect in this theory is the coupling of microrotation vector with the microtemperatures even for isotropic bodies. We note that in the classical theory of Cosserat thermoelasticity for isotropic bodies, the microrotation vector is independent of the thermal field. In Section 2, we establish the field equations of the linear theory of thermoelasticity with microtemperatures. A uniqueness theorem in the dynamical theory of anisotropic bodies is presented in Section 3. In Section 4, we study the continuous dependence of solutions upon initial data and body loads. Section 5 is concerned with the effects of a concentrated heat source in a body that occupies the entire three-dimensional euclidean space.

2. Field equations

In the first part of this section we present the general balance laws of a continuum with microstructure in the form given by Grot (1969) and Eringen (1999). Then we derive the field equations of the linear theory of microstretch thermoelastic bodies with microtemperatures. The second moment of stress tensor and the microstress moment average are neglected in the balance laws since these functions appear only nonlinearly in the field equations (cf. Grot, 1969).

We consider a body that at some instant occupies the region B of the euclidean three-dimensional space and is bounded by the piecewise smooth surface ∂B. The motion of the body is referred to a fixed system of rectangular cartesian axes Ox_i $(i=1,2,3)$. We denote by $\mathbf{n}$ the outward unit normal of ∂B. Boldface characters stand for tensors of an order $p \geqslant 1$, and if $\mathbf{v}$ has the order p, we write $v_{ij\ldots s}$ (p subscripts) for the components of $\mathbf{v}$ in the cartesian coordinate frame. We shall employ the usual summation and differentiation conventions: Latin subscripts are understood to range over the integers $(1,2,3)$, summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding cartesian coordinate. In what follows we use a superposed dot to denote partial differentiation with respect to the time t.

Let $\mathbf{u}$ be a displacement vector field over B. The balance of linear momentum can be written in the form

$$t_{ji,j} + \rho f_i = \rho \ddot{u}_i, \tag{2.1}$$

where t_{ij} is the stress tensor, ρ is the reference mass density, and f_i is the body force. We denote by m_{ijk} the first stress moment tensor. The balance of first stress moments is given by

$$m_{kij,k} + t_{ji} - s_{ji} + \rho \ell_{ij} = \rho \dot{\sigma}_{ij}, \tag{2.2}$$

where s_{ij} is the microstress tensor, ℓ_{ij} is the first body moment density and σ_{ij} is the inertia per unit mass. Let e be the internal energy density per unit mass, and let ε_i denote the first moment of energy vector. The balance of energy and the balance of first moment of energy can be expressed as

$$\rho \dot{e} = t_{ij} v_{j,i} + (s_{ij} - t_{ij}) v_{ji} + m_{kij} v_{ij,k} + q_{j,j} + \rho S \tag{2.3}$$

and