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Stress intensity factor computation along a non-planar curved crack in three dimensions

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Abstract

In this paper, a numerical procedure, incorporated with the finite element method, is developed for calculation of the mixed-mode stress intensity factors along a 3D curved crack with non-planar surfaces. The approach is formulated by modifying the concept of the J_k- and G_{III}-integrals. Note that the property of path-independence for curved cracks does not hold in a manner as that for straight cracks and needs to be properly modified. As a consequence, the near-front region is always included in the integration. An appropriate numerical method is therefore developed for simulating the associated singular behavior. Since no extra auxiliary solutions are required for the computation, the proposed method appears to be versatile and can be used for evaluation of the stress intensity factors associated with crack front and crack surfaces of arbitrary curvatures.

Keywords: Stress intensity factors; Curved crack; Non-planar crack surfaces; J_k- and G_{III}-integrals; Modified surface-independence; Singular behavior

1. Introduction

The crack geometry is one of the major factors that influences the overall resistance for many engineering structures. For two-dimensional problems, the crack may be straight or curved. In three dimensions, the crack front may be either straight or curved, and also the crack surfaces may be non-planar. The fracture behavior associated with three-dimensional cracks thus depends on both the crack front curvature and the crack surface curvature. Numerous analytical studies on description of the mixed-mode asymptotic stress behavior for some special cases of curved shapes for either planar or non-planar cracks have been performed (e.g., Sih et al., 1962; Cotterell and Rice, 1980; Dreilich and Gross, 1985; Chen, 1999, etc.). Nevertheless, more detailed investigations on the near-tip singular behavior for cracks of general curved shapes are still in need.

The mixed-mode singular near-tip stress field for a crack tip in linear elasticity can be effectively characterized by the stress intensity factors (SIFs). Direct evaluation of the SIFs with numerical schemes such as finite

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element method appears to be difficult due to the complicated mechanical state around the singular point. Alternatively, the SIFs for straight cracks (in two dimensions) and cracks with planar surfaces (in three dimensions) can be determined by using various types of contour and surface (or domain) integrals, e.g., the J_k-integrals (Knowles and Sternberg, 1972; Budiansky and Rice, 1973), interaction integral (Stern et al., 1976), M_1-integral (Chen and Shield, 1977), and domain integral (Nikishkov and Atluri, 1987; Shivakumar and Raju, 1992), etc. While most of these integrals are based on similar concept, they were developed with the appearance of varying forms. Note that, when mixed-mode loading conditions are considered, the asymptotic singular behavior is always inevitably included in evaluating any of the aforementioned integrals.

As to curved cracks with non-planar surfaces, the conventional concept of the above integrals is no longer valid and needs to be modified. A number of attempts have therefore been proposed for calculation of fracture parameters associated with such crack geometry. For example, for two-dimensional problems, the SIFs have been evaluated by using contour integrals (accompanied by a surface integral) for cracks of circular arc (Lorentzon and Eriksson, 2000) and various types of curvature (Chang and Wu, 2006). Also, boundary integral (Sladek and Sladek, 1983), surface integral (Forth and Keat, 1996), hypersingular boundary integral (Dominguez and Ariza, 2000), interaction energy integral (Gosz and Moran, 2002), and domain integral (Eriksson, 2002) were developed for applications containing three-dimensional cracks. It should be noted that, in order to perform these integrals, proper auxiliary solutions associated with the prescribed crack geometry are usually required. Therefore, to the authors' knowledge, no general formulation associated with finite element method for problems containing non-planar cracks of arbitrary curved shapes has been presented.

In this paper, a numerical procedure is developed for calculation of the mixed-mode stress intensity factors for three-dimensional cracks where both the front and the surfaces may be curved. The approach is based on the concept of the J_k- and G_{III}-integrals. By comparing with the other types of integrals, J_k and G_{III} appear to be attractive in practice since no extra auxiliary solutions are required in their formulation. With this superiority, they can thus be generally applicable for problems of varieties of curved shapes. Nevertheless, the singular mechanical behavior in the near-front region is always involved in the calculation and special attention is therefore addressed for accurate solutions. No particular singular elements are used in the calculations.

2. The surface integrals for curved cracks

Consider a crack in a three-dimensional homogeneous elastic body (Fig. 1), where both the front and the surfaces are of arbitrary curved shape. We introduce a local coordinate system originating at a point O along the crack front. The x_2-axis is perpendicular to the tangential plane of the crack surface, while the x_1- and x_3-axes lie in this plane and are normal and tangent, respectively, to the crack front. Note that expressions for the asymptotic mixed-mode stresses in the immediate neighborhood of the crack tip O are the same for cracks with either planar or non-planar surfaces in that they are applied in the near-tip region where the crack front lying asymptotically along the x_3-axis and the crack surfaces lying on the x_1–x_3 plane.

The J_k-integral was originally defined in two dimensions as contour integrals taken around a singular point. For a curved crack in three dimensions, by taking a curved tubular surface S_i of radius ρ around the near-tip part of the crack front (Fig. 2a), the pointwise J_k-integrals associated with crack tip O are defined over S_i as

$$J_k = \lim_{\substack{\rho/L\to 0 \\ L\to 0}} \int_{S_i} \left[W n_k - \sigma_{mj} n_j \left(\frac{\partial u_m}{\partial x_k} \right) \right] \mathrm{d}a, \quad k = 1,2 \tag{1}$$

where x_k are the Cartesian components of the local curvilinear coordinate system, W is the strain energy density of the material, u_m are the Cartesian components of the displacement vector ($m = 1,2,3$), n_j are the Cartesian components of outward unit vector normal to S_i ($j = 1,2,3$), a is the area over the surface, and L is the arc length in the curvilinear x_3-direction. Transverse section of the surface in the x_1–x_2 plane is shown in Fig. 2b, where (r,θ) denote the polar components of the local coordinate. Still, Eq. (1) is valid for both planar or non-planar cracks in that the surface of integration, S_i, is defined as a tube encircling the portion of crack front and shrinking onto the tip O (this limiting case is not shown in Figs. 2a and 2b).

Physically, the first component of J_k (i.e., J_1) is identified as the pointwise energy release rate at point O due to crack advancing along its original orientation (i.e., x_1-axis) and is often employed as a critical energy