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Perturbation methods for the analysis of the dynamic behavior of damaged plates

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Abstract

This paper presents an analytical method for the analysis of the dynamic behavior of damaged plates. The proposed approach allows the derivation of mode shapes and corresponding curvature modes for plates with various kinds of defects. Damage is modeled as a localized reduction in the plate thickness. Both point and line defects are considered to model notches or line cracks and delaminations in the plate. Small thickness reductions are considered so that the dynamic behavior of the damage plate can be analyzed through perturbations with respect to the undamaged modes. Results are presented to demonstrate the sensitivity of the curvature modes with respect to the considered low damage levels. Also, the curvature modes are used for the estimation of the strain energy of the plate and for the formulation of a damage index which can be used to provide damage location and extent information.

Keywords: Perturbation methods; Strain energy ratio; Damaged plates; Curvature modes

1. Introduction

The objective of a structural health monitoring system is to identify anomalies or damages such as cracks, delamination and disbonds in structures. The term identification includes the determination of the existence of damages, their location and their sizes or magnitudes as accurately as possible. This goal can be achieved with the help of analytical formulations for simple structures, which can provide invaluable insight in the interpretation and analysis of the measured dynamic response of structures under investigation.

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The effects of cracks on the dynamic behavior of beams and shafts have been studied by many authors. Excellent overviews of the state-of-the-art can be found for example in Staszewski et al. (2004) and Doebling et al. (1996). The analytical modeling of simple beam structures with through-the-width cracks has also engaged many researchers. Most of the existing formulations are based on the description of damage as an equivalent stiffness at the location of the defect. The dynamic behavior of single-sided cracked beams can for example be found in Gudmundson (1984), Ostachowicz and Krawczuk (1990), Krawczuk (2002a); Krawczuk and Ostachowicz (2002) and Krawczuk and Ostachowicz (1995), while the work of Hellan (1984), Atluri (1986) and Haisty and Springer (1988) analyzed the effect of double-sided cracks of equal depth. A different approach of modeling cracked beams consists in using approximated numerical solutions. For example in Christides and Barr (1984), the variation of the fundamental frequency of a simply supported beam with a mid-span crack is evaluated using a two-term Rayleigh–Ritz solution. In the approximation an exponentially decaying crack function was used to simulate damage, and the decay rate of the function was estimated from experimental results. The Galerkin approximation is used alternatively in Shen and Pierre (1990) in order to achieve fast convergence rates, while in Qian et al. (1991), a Finite Element model is used to predict the behavior of a beam with an edge crack. Finally, Luo and Hanagud (1998) and subsequently Lestari (2001) presented a perturbation method to describe the dynamic behavior and in particular the curvature modes of cracked beams. In these works, the perturbation analysis is based on the assumption of a small crack whose depth is defined by a perturbation parameter. The modal behavior of the cracked beams is evaluated through perturbation of the modal parameters of the undamaged beams, so that approximate analytical expressions for the damaged modes can be obtained. The present paper extends the formulation presented in Luo and Hanagud (1998) and Lestari (2001) to plates with localized defects. Both point defects, or notches, as well as line defects are considered to evaluate their effects on natural frequencies, mode shapes and curvatures. Relatively little work can be found in the literature on the analytical modeling of damaged plates. Among the work considered as reference for this study, the contributions by Ostachowicz, Krawczuk and co-workers are here mentioned as relevant to the present investigations (Azak et al., 2001; Krawczuk, 2002b; Krawczuk et al., 2004).

The analytical formulations presented below can be used in support of experimental tests, to analyze data and to supplement the experiments with mechanics-based analysis tools that quantify damage. In particular, the application of scanning laser vibrometry for the detection of dynamic deflection shapes allows unprecedented amounts of information which can be successively used for the evaluation of curvature shapes. The results presented in this work and in Luo and Hanagud (1998) and Lestari (2001), in fact indicate how curvatures are extremely sensitive to damage, and how they can be successfully utilized as part of a damage detection technique.

The paper is organized as follows. The brief introduction given in this section is followed by the presentation of the analytical formulation and of the perturbation analysis for damaged plates given in Section 2. Section 3 then presents results for plates with point defects or notches and discusses the influence of defect size and location on various modal parameters. Section 4 extends the investigation to line defects of various orientation, size and location, while Section 5 presents initial experimental investigations on damaged plates. Finally, Section 6 summarizes the main results of the work and outlines the directions of future research activities.

2. Dynamics of damaged plates

2.1. Equation of motion

The free dynamic behavior of damaged plates can be described by expressions formulated from the general equation of motion for plates of variable thickness, as found in Leissa (1993):