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Eshelby tensors for a spherical inclusion in microstretch elastic fields

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Abstract

In the present work, microelastic and macroelastic fields are presented for the case of spherical inclusions embedded in an infinite microstretch material using the concept of Green's functions. The Eshelby tensors are obtained for a spherical inclusion and it is shown that their forms for microelongated, micropolar and the classical cases are the proper limiting cases of the Eshelby tensors of microstretch materials.

Keywords: Eshelby tensor; Eigenstrain; Microstretch; Microelongation; Micropolar; Green's functions

1. Introduction

A microcontinuum is considered as the collection of material particles, which can deform independently in the microscale in addition to the classical bulk deformation of the material. Eringen and Suhubi (1964) and Suhubi and Eringen (1964) introduced and developed a general theory for this phenomenon which is called micromorphic continua. As it is known, the general micromorphic theory is very complicated even for the linear case. To overcome the difficulties, Eringen introduced first the micropolar elasticity (Eringen, 1966) and, then the microstretch elasticity (Eringen, 1990). Because of their well suitability to the nature of many materials, both theories were universally accepted.

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Although the numbers of the unknown constitutive coefficients in micropolar and microstretch theories are considerably less than the general case, there are still some undetermined constitutive coefficients. Thus, we followed a different approach (Inan, 1990) and considered an additional homogenization procedure similar to the one known for the composite materials, to evaluate and estimate overall effective material properties. To apply the usual homogenization techniques (Mori–Tanaka method (Mori and Tanaka, 1973; Benveniste, 1987), for instance), we need to know the Eshelby tensors which establish the relation between the strains of the matrix material and of the inclusion. Several problems have been solved by using the Eshelby's equivalent inclusion theory (Eshelby, 1957). For the linear theory of the asymmetric elasticity, some solutions are given by Sandru (1966). Hsieh et al. (1980) and Hsieh (1982) have derived general formulas for the volume defects in micropolar media. Finally, Cheng and He (1995, 1997) obtained four Eshelby tensors for the spherical and the circular cylindrical inclusion in an isotropic centrosymmetric micropolar media, respectively. Sharma and Dasgupta (2002) calculated averaged stress and strains using numerical versions of the micropolar Eshelby tensors and extended the Mori–Tanaka method to the micropolar medium.

In the present work, the fundamental solutions are obtained for the microstretch medium. And then the Eshelby tensors are obtained for a spherical inclusion and it is shown that the Eshelby tensors for the microelongated, micropolar and the classical cases are the limiting cases of the Eshelby tensors of microstretch materials.

2. Fundamental solutions

The fundamental solutions for microelongated and micropolar media are given by Kiris and Inan (2005) and Cheng and He (1995), respectively. Applying a similar method given in Kiris and Inan (2005), we obtain the fundamental solutions and then the Eshelby tensors for microstretch medium.

As it is mentioned in the introduction, microstretch material is defined as the body with non-rigid particles which may do volume changes and microrotations in addition to the bulk deformation in the microstructural level. In other words, the material particles of such a material can stretch and contract independently of each others translations and rotations.

Since our task is to obtain the Eshelby tensors for the microstretch materials, first we shall obtain the field equations of the medium. The local forms of the equations of balance of momentum and moment of momentum at a point of a deformed microstretch body for the static case are given as (Eringen, 1999)

$$\begin{aligned} &t_{kl,k} + f_l = 0, \\ &m_{kl,k} + \in_{lmn} t_{mn} + l_l = 0, \\ &m_{k,k} + t - s + l = 0, \end{aligned} \tag{1}$$

where t_{kl} is the stress tensor, s_{kl} and m_{kl} are the couple stress tensors, m_k is the microstretch vector and $t = t_{kk}$, $s = s_{kk}$. f_l, l_l and l are the body force, the body moment and the body force densities, respectively. Subscripts preceded by a comma stand for derivatives with respect to the corresponding spatial coordinates and $\in_{lmn}$ is the permutation symbol.

The geometrical definitions and relations are given below:

$$\varepsilon_{kl} = u_{l,k} + \in_{lkm} \phi_m, \quad \gamma_{kl} = \phi_{k,l}, \quad \gamma_k = 3\theta_{,k}, \quad e = 3\theta. \tag{2}$$

Here, ε_{kl} is the strain tensor, ϕ_k is the microrotation vector, θ is the microelongation and u_k is the displacement vector.