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An elastic–plastic crack bridging model for brittle-matrix fibrous composite beams under cyclic loading

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Abstract

A fibrous composite beam with an edge crack is submitted to a cyclic bending moment and the crack bridging actions due to the fibers. Assuming a general elastic-linearly hardening crack bridging model for the fibers and a linear-elastic law for the matrix, the statically indeterminate bridging actions are obtained from compatibility conditions. The elastic and plastic shake-down phenomena are examined in terms of generalised cross-sectional quantities and, by employing a fatigue crack growth law, the mechanical behaviour up to failure is captured. Within the framework of the proposed fracture mechanics-based model, the cyclic crack bridging due to debonding at fiber–matrix interface of short fibers is analysed in depth. By means of some simplifying assumptions, such a phenomenon can be described by a linear isotropic tensile softening/compressive hardening law. Finally, numerical examples are presented for fibrous composite beams with randomly distributed short fibers.

Keywords: Bridged crack; Brittle-matrix fibrous composite beam; Elastic–plastic model; Fatigue crack growth; Linear hardening/softening; Cyclic loading

1. Introduction

As is well-known, by incorporating ductile fibers into the brittle matrix of a composite material, several mechanical properties can be improved (cracking resistance, ductility, impact resistance, fatigue strength). Fiber-reinforced cementitious composites are employed in an increasing amount of civil engineering structures. These materials under cyclic loading tend to develop cracks in the matrix, and such cracks are

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Nomenclature

a	crack depth
b	height of the beam cross-section
c_i	position of the ith reinforcement with respect to the bottom of the beam cross-section
D_f	fiber diameter
e_i	elastic part of crack opening translation at the ith reinforcement level
E	Young modulus of the matrix
E_f	Young modulus of the fibers
F_i	crack bridging force of the ith reinforcement
$F_{P,i}$	initial yield force of the ith reinforcement
$\overline{F}_{P,i}$	current yield force of the ith reinforcement
h_i	hardening modulus of the crack bridging law for the ith reinforcement
$K_{0,i}$	elastic stiffness of the crack bridging law for the ith reinforcement
$K_{t,i}$	plastic stiffness of the crack bridging law for the ith reinforcement
K_I	stress intensity factor
K_{IC}	critical stress intensity factor (fracture toughness)
l	embedded length of a single fiber
L_f	fiber length
M	bending moment
M_F	bending moment of either unstable fracture or crushing of the matrix
M_P	plastic bending moment
M_{SD}	shake-down bending moment
n	number of reinforcements intersected by the crack
N	number of loading cycles
p_i	plastic part of crack opening translation at the ith reinforcement level
P	pull-out force of a single fiber
P_P	initial yield pull-out force of a single fiber (peak load)
t	thickness of the beam cross-section
V_f	fiber volume fraction
w_i	crack opening translation at the ith reinforcement level
β	load factor
δ	pull-out translation of a single fiber
δ_P	pull-out translation of a single fiber at the initial yield pull-out force P_P
$\zeta_i = c_i/b$	relative position of the ith reinforcement with respect to the bottom of the beam cross-section
κ_i	hardening parameter of the crack bridging law for the ith reinforcement
λ_{ij}	localised compliance related to the crack opening translation at the ith reinforcement level due to a unit crack opening force $F_j = 1$ acting at ζ_j
λ_{iM}	localised compliance related to the crack opening translation at the ith reinforcement level due to a unit bending moment $M = 1$
λ_{MM}	rotational localised compliance due to a unit bending moment $M = 1$
$\xi = a/b$	relative crack depth
π_i	plastic part of crack opening translation at the ith reinforcement level, accumulated along the tensile or compressive direction
σ_P	initial yield crack bridging force per unit crack surface