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Postbuckling analysis of elastic shells of revolution considering mode switching and interaction

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Abstract

The postbuckling response of shells is known to exhibit complex phenomena including mode switching and interaction, particularly in the advanced postbuckling range. The existing literature contains many initial postbuckling analyses as well as advanced postbuckling analyses for a single buckling mode, but little work is available on the advanced postbuckling analysis of shells of revolution considering mode switching and interaction. In this paper, a numerical method for the advanced postbuckling analysis of thin shells of revolution subject to torsionless axisymmetric loads is presented, in which such mode switching and interaction are properly captured. Numerical results obtained using the present method for several typical problems not only demonstrate the capability of the method, but also lead to significant observations concerning the postbuckling behavior of thin shells of revolution. In particular, the results show that strong interaction between different harmonic modes may exist and the transition of deformation mode from one to another is gradual. Consequently, the conventional approach of finding the postbuckling path of a shell as the lower festoon curve of postbuckling paths of individual harmonic modes is not valid and is at best a convenient approximation.

Keywords: Shells; Shells of revolution; Postbuckling; Mode switching; Mode interaction

1. Introduction

Axisymmetric shells are widely used in many engineering fields. Examples include aircraft, spacecraft, submarines, nuclear reactors, cooling towers, storage silos and tanks, roof domes, offshore platforms,

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tubular towers, chimneys, pressure vessels and pipelines. A perfect shell of revolution under axisymmetric loads may bifurcate into a non-symmetric mode at a suitable level of loading. The load carrying capacity of a corresponding real shell depends not only on the bifurcation load, but also on the nature of the post-bifurcation path, which determines the sensitivity of the shell to initial geometric imperfections. As a result, the postbuckling behavior of perfect shells has been of enormous interest to shell stability researchers and designers. Nevertheless, postbuckling analysis of perfect shells has been and still remains a challenge to numerical analysts, because the postbuckling behavior of perfect shells may be highly unstable and very complicated, and may involve complex mode switching and interaction. It should be noted that some of the difficulties encountered in the postbuckling analysis of perfect shells abate or disappear when significant geometric imperfections of a suitable form are included in the analysis. In this sense, an analysis of the postbuckling behavior of perfect shells is more challenging than a nonlinear analysis of imperfect shells for which the phenomena of bifurcation and mode switching are likely to be eroded by the presence of significant geometric imperfections. This paper is concerned only with perfect shells, whose postbuckling behavior is important in its own right and for a proper understanding of the sensitivity of the shell to geometric imperfections.

The existing literature on postbuckling behavior contains many initial postbuckling analyses (e.g. Budiansky and Hutchinson, 1966; Koiter, 1945). A number of researchers have also implemented the general theory of initial postbuckling in finite element analyses of axisymmetric shells (e.g. Azrar et al., 1993; Endou et al., 1976; Flores and Godoy, 1992, 1993). However, these studies were able to predict only the initial part of the postbuckling path. Rigorous analytical studies (e.g. Esslinger and Geier, 1975; Shen, 1996; Shen and Chen, 1991; Yamaki, 1984) have also examined advanced postbuckling responses, but these analyses were concerned with postbuckling deformations in a single buckling mode, so mode switching and interaction were not considered. More recently, Bulenda (1993) presented a harmonic-by-harmonic finite element method to compute the postbuckling path of a cylindrical shell under uniform external pressure, but mode switching was again not considered. Similarly, Combescure's (1999) results for the postbuckling of an elastic–plastic shell with significant imperfections under external pressure were not concerned with mode switching. He did not observe any mode change in his static results, but noticed such a change in his dynamic results and attributed this to inertial effects. The best investigation so far appears to be that by Kato et al. (1997) who studied the secondary postbuckling behavior and mode interaction in spherical caps under uniform external pressure. Although secondary bifurcation was considered in their study by monitoring the eigenvalue of a chosen secondary bifurcation mode, the choice of this mode appears to be arbitrary apart from the exclusion of those describing the primary post-bifurcation path. As a result, mode switching was not properly captured. Only spherical caps with the first and second bifurcation points being almost coincidental were studied, so the capability of the method for other situations is not clear.

The modelling of mode switching is very challenging, and there have been a number of recent attempts using general shell elements either employing a static approach or a dynamic approach (Choong and Ramm, 1998; Guggenberger, 1996; Kusher, 1997; Riks et al., 1996). These studies are still exploratory, with only limited successes. There appears to have been no previous study using axisymmetric shell elements which properly modelled the advanced postbuckling deformation process of perfect elastic shells of revolution involving continuous mode switching and interaction.

In this paper, a numerical method for the advanced postbuckling analysis of elastic shells of revolution subject to torsionless axisymmetric loads is presented, based on the general formulation for nonlinear non-symmetric deformations of shells of revolution presented in Hong and Teng (2002). The method is based on the use of small load-disturbances in a nonlinear analysis of shells of revolution under general non-symmetric loads, and is referred to as the load-disturbance method. By specifying small load-disturbances in appropriate harmonic modes, complex postbuckling paths with mode switching and interaction can be predicted. Numerical results of a number of typical problems solved using this method are presented to demonstrate its accuracy and capability.