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Chaotic vibration of a nonlinear full-vehicle model

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Abstract

The present work investigates the chaotic responses of a nonlinear seven degree-of-freedom ground vehicle model. The disturbances from the road are assumed to be sinusoid and the time delay between the disturbances is investigated. Numerical results show that the responses of the vehicle model could be chaotic. With the bifurcation phenomenon detected, the chaotic motion is confirmed with the dominant Lyapunov exponent. The results can be useful in dynamic design of a vehicle.

Keywords: Chaos; Nonlinear dynamics; Ground vehicle

1. Introduction

Since the vehicle dynamics is concerned with controllability and stability of automobile, it is important in design of a ground vehicle. The modeling of the vehicle with the analysis of the dynamic response of the mathematical model have been examined in a large number of previous investigations (Genta, 2003; Takahashi, 2003). In these studies, three typical models have been developed with researches related to the dynamic behavior of vehicle and its vibration control. The simplest representation of a ground vehicle is a quarter-car model with a spring and a damper connecting the body to a single wheel, which is in turn connected to the ground via the tire spring (Robson, 1979; Williams, 1997; Yang et al., 2001). As shown in Fig. 1(a), the mass representing the wheel, tire, brakes and part of the suspension linkage mass, is referred to as the unsprung mass. The quarter-car model is used only when the heave motion needs to be considered.

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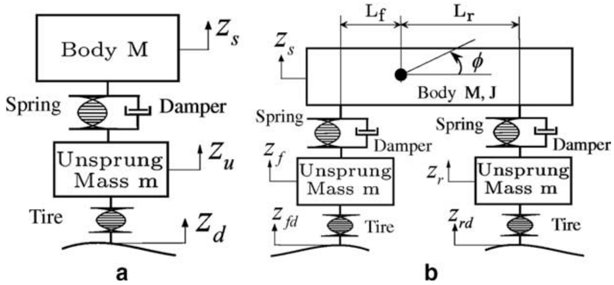

Fig. 1. Simplified vehicle model: (a) quarter-car model and (b) half-car model.

A half-car model is shown in Fig. 1(b). It is a two wheel model (front and rear) for studying the heave and pitch motions (Moran and Nagai, 1994; Vetturi et al., 1996; Campos et al., 1999). This four degree-of-freedom model allows the study of the heave and pitch motions with the deflection of tires and suspensions. Comparing to the full 3-D vehicle model, the half-car model is relatively simple to analyze and yet can reasonably predict the response of the system (Oueslati and Sankar, 1994). Therefore many researchers often use it. A more complex model is the full vehicle model which is a four wheel model with seven degree-of-freedom done for studying the heave, pitch and roll motions (Ikenaga et al., 2000).

In the studies of dynamic response of ground vehicle using these mechanical models, the spring and damper are usually assumed to be linear components for simplification. However, in practice an automobile is a nonlinear system because it consists of suspensions, tires and other components that have nonlinear properties. Therefore, the chaotic response may appear when the vehicle moves over a bumpy road. Since the chaotic responses is a random like motion, it could be harmful to the vehicle. By the lack of theoretical tool for predicting parameters in a system which induce chaotic response, the study of chaotic response for specified mechanical model is still needed. The main objective of the present study is investigate the chaotic response in a nonlinear seven degree-of-freedom vehicle. In Section 2, a nonlinear seven degree-of-freedom ground model is introduced and the motion equations of the system are derived. Next, the numerical simulation is conducted and the dynamic responses of the model are discussed. Frequency response diagrams, bifurcations and Poincaré maps are used to trace chaotic motion of the system. The dominant Lyapunov exponent is used to identify the chaos. The results indicate that the chaotic vibration exist as the forcing frequency is in the unstable region of the frequency response diagram of the system.

2. Simulation model

Fig. 2 shows the nonlinear full-vehicle model: the vehicle body is represented by a three degree-of-freedom rigid cuboid with mass m_s. The heave, pitch and roll motions of the sprung mass are considered. The four unsprung masses (front-left, front-right, rear-left and rear-right) are connected to each corner of the rigid cuboid. It is assumed that the four unsprung masses are free to bounce vertically. The suspensions between the sprung mass and unsprung masses are modeled as nonlinear spring and nonlinear dampers elements, while the tires are modeled as nonlinear springs with viscous damping.

It is assumed that the modeled nonlinear spring of suspension has following characteristics:

$$F_{sij} = k_{sij}\,\mathrm{sgn}(\Delta_{sij})|\Delta_{sij}|^{n_{sij}} \quad (i = f, r,\ j = \ell, r), \tag{1}$$

where F_{sij} is the spring dynamic force, k_{sij} is the equivalent stiffness, Δ_{sij} is the deformation of the spring and $\mathrm{sgn}(\cdot)$ is the signum function. The subscript s means spring of suspension, the subscript $i = f$, r indicates the front and rear while the subscript $j = \ell$, r, indicates left and right. This way, F_{sfr}, k_{sfr} and Δ_{sfr} indicate the