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Mode-3 spontaneous crack propagation in unsymmetric functionally graded materials

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Abstract

The effects of the inhomogeneous material property variation on the mode-3 crack propagation characteristics have been analyzed. The spectral form of the elastodynamic boundary integral equations are derived for a functionally graded material, which is used as the numerical tool in our analysis. The material property gradient is assumed to vary unsymmetrically in the direction normal to the fracture plane. A parametric study has been performed by systematically varying the inhomogeneity length scale. In comparison with a homogeneous material, an unsymmetric functionally graded material offers lesser fracture resistance. The fracture resistance progressively decreases with increase in inhomogeneity, quantified by the increase in crack sliding displacement jumps, crack tip velocities and accelerations. The material property inhomogeneity affects only the transient crack propagation velocities, while the quasi-steady-state velocity remains unaltered.

Keywords: Functionally graded materials; Multi-functional materials; Inhomogeneity length scale; Anti-plane shear; Spontaneous crack propagation; Spectral scheme

1. Introduction

Many biological structures such as bones and shells have material properties that vary smoothly with spatial position in order to efficiently respond to the surrounding mechanical loads (Krassig, 1993; Suresh and Mortensen, 1998). The concept of tailoring the spatial properties in engineering is not new as well, for example case hardened steel components have hardness varying continuously with depth in order to provide better wear resistance. Also, composites both layered as well as particulate have reinforcing material in either specific directions or at random to increase the specific strength and stiffness of the structure. One draw back of the composite systems is that there exists a distinct interface across the plies or the particle-matrix, which acts as a source of stress concentration and a potential site for delaminations or cracks to initiate. The concept of a functionally graded material tries to resolve the problem of the material property mismatch by continuously

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varying them as a function of the spatial coordinates. For the purpose of this study, a functionally graded material (FGM) is an inhomogeneous material with a known functional form of the material inhomogeneity. Applications such as thermal barrier coatings (Movchan and Yakovchuk, 2004), ballistic impact resistance structures (Chin, 1999) and wear resistive coating (Schulz et al., 2003) are some examples where FGMs are used. Modern day structural designs are based on the concepts of fracture mechanics and a thorough understanding of the fracture response of FGMs is key in designing structures using them. Hence, the primary objective of this study is to understand the fracture response of graded materials. In particular, the effect of material inertia is considered here, that can become either important when the loads applied are time dependent or even when the velocity of crack propagation becomes comparable to the material wave speed.

One of the early works in fracture mechanics considering the material inhomogeneity was that of Delale and Erdogan (1983), who performed an asymptotic analysis of a stationary tensile crack and extracted the stress-intensity factor. From their analysis they found that the near-tip stress-intensity factor was not affected by the material property inhomogeneity. Eischen (1987) in his asymptotic stress-series (similar to the Williams (1952) expansion for homogeneous materials) confirmed that the most singular term was unaffected by the inhomogeneous material property variation. Parameswaran and Shukla (2002) have obtained the higher order terms in the asymptotic stress-series expansion and found that the material inhomogeneity affects only the higher order terms. Zhang et al. (2003) have studied the effect of material inhomogeneity on the mixed-mode stress-intensity factor ahead of a stationary crack. Due to the complexity of the analysis, only a limited number of studies available in the literature have considered the effect of material inertia in functionally graded materials. Parameswaran and Shukla (1999) have obtained the asymptotic stress and displacement fields for a crack propagating in a graded material. Their analysis assumed exponential and linear variation of the properties in the continuum. Meguid et al. (2002) have studied the effect of material property inhomogeneity on the crack speed variation and dynamic stress-intensity factor in a FGM. They solved a singular integral equation in their analysis, which indicated that the material inhomogeneity has no effect on the most singular term. Recently, Wang and Nakamura (2004) have performed rapid crack propagation simulations in FGMs using a finite element method. They have varied the fracture toughness in the direction of the crack propagation by continuously varying the cohesive zone properties. Their study showed that the crack propagation characteristics were affected by the material property inhomogeneity. In the present study, we propose to understand the effects of material inhomogeneity and inertia by performing numerical simulations of rapid crack propagation in functionally graded materials. There are several robust numerical techniques commonly used to simulate rapid crack propagation in solids, this includes, boundary integral method (Israil and Banerjee, 1990; Liu and Rizzo, 1993), finite differences (Yang and Ravi-Chandar, 1996; Mikumo et al., 1987), finite elements (Wang and Nakamura, 2004; Atluri and Nishioka, 1985; Safjan and Oden, 1993) and cohesive volume finite element (CVFE) (Xu and Needleman, 1994; Geubelle and Baylor, 1998; Camacho and Ortiz, 1996). All of the above-mentioned techniques are computationally intensive and mostly suited for simulations when the crack propagation path is not known a priori. The focus of the present work is to understand the effect of the material property inhomogeneity on the crack propagation characteristics. Hence, we restrict our attention to simulating planar, rectilinear crack propagation. Geubelle and Rice (1995) have developed an efficient numerical tool called the "spectral scheme" that can simulate planar crack propagation. The scheme is versatile and can handle a variety of applied tractions (Geubelle and Kubair, 2001), state- and rate-dependent cohesive (Kubair et al., 2002) and friction models (Rice et al., 2001). The spectral scheme has been derived for various material systems such as bimaterial interfaces (Geubelle and Breitenfeld, 1997; Breitenfeld and Geubelle, 1998), viscoelastic materials (Geubelle et al., 1998), and orthotropic solids (Hwang and Geubelle, 2000). In this article, we extend the spectral formulation to simulate spontaneous crack propagation in functionally graded materials under anti-plane shear loading conditions. The formulation and the details of the numerical implementation are presented in Sections 2 and 3, respectively. In Section 4, we present the results from our parametric study that we have performed using the spectral scheme and illustrate the effect of the material property inhomogeneity on the crack propagation characteristics.

2. Spectral formulation

The spectral form that will be presented here is an exact elastodynamic relation between the Fourier coefficients of the tractions and the corresponding displacement discontinuities. As mentioned earlier, the spectral