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# On variational approaches in NRT continua

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**Abstract**

In the paper some features of the theory of Not Resisting Tension (NRT) material are deepened. In details, one first introduces the basic NRT model, which is proved to simply and effectively interpreting the behaviour of mechanical bodies made by not-cohesive materials; thereafter one analyses energetic approaches and limit analysis tools for problems relevant to NRT continua. Afterward, on the basis of the fundamental variational theorems, the main rules governing the NRT behaviour are demonstrated, by imposing Kuhn–Tucker stationarity conditions for the stated constrained optimisation procedures. Finally an application is operated of the presented theory to an elastic NRT semi-plane subject to a distributed load, reproducing the stress situation induced in the soil by a foundation structure.


*Keywords:* Bi-dimensional statics; NRT model; Foundation structures; Energetic approach; Stress field; Stress propagation

## 1. The NRT material

### 1.1. General features of NRT behaviour

The *Not Resisting Tension* (NRT) material (Baratta, 1991; Di Pasquale, 1984; Bazant, 1996a) is a simple and complete phenomenological model for interpreting the mechanical bodies made by not-cohesive materials.

The theory of the Not Resisting Tension material, originally formulated by Heyman (Heyman, 1966; Heyman, 1969), is based on two fundamental hypotheses: the assumption of zero tensile resistance and the hypothesis of linear elastic behaviour in compression.

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Even as an idealization, the NRT model gives a pretty reliable representation of the real behaviour of materials exhibiting light tensile resistance, such as masonries or soils (Baratta, 1984, 1996; Cambou and Di Prisco, 2002; Broberg, 2000; Li and Bazant, 1996; Bazant, 1995; Acker et al., 1998).

The NRT material is usually conceived by assuming an elastic response, possibly non linear, along planes characterized by pure compression stresses, while one admits the development of free deformations (*fracture strains*) without energy dissipation along the other directions. That is to say that the NRT material is essentially a non-linear elastic material, whose non-linearity is mainly due to the development of fractures, which are absent in the compressive phase.

The NRT constitutive law is, then, usually schematised by an elastic stress–strain relation, which is unilateral, that is to say valid with reference to purely compressive fields.

Therefore, the domain of admissible stresses coincides with the Rankine's square, with infinite limit tension in compression.

By assuming negative values for compressive stresses, at the generic point $P$ (Fig. 1) the stress tensor $\boldsymbol{\sigma}$ should be characterized by stress values along the principal directions '1' and '2' satisfying admissibility conditions; that is to say that, imposing $\sigma_2 \leqslant \sigma_1$, admissible principal stress states should satisfy the condition $\sigma_1 \leqslant 0$.

The admissibility condition can be equivalently expressed with reference to the generic plane elements $\pi_a$ normal to the lines '*a*' passing trough the point $P$, by considering the normal $\sigma_a$ and tangential $\tau_a$ components of the stress vectors $\mathbf{t}_a$; in this case it imposes that $\sigma_a \leqslant 0$ with $\tau_a$ undefined on $\pi_a$.

Since the material is not able to resist tensile stresses, it is necessary to allow for the development of inelastic strain $\boldsymbol{\varepsilon}_\mathrm{f}$ (*fracture strain* tensor) superposing to the elastic strain $\boldsymbol{\varepsilon}_\mathrm{e}$.

The fracture strain has the role of transferring those forces deriving from inadmissible tensile stresses to the neighbouring material, in cases where the body has the capacity of achieving equilibrium with the same forces in pure compression.

At any point where the fracture tensor is not zero, contact at the inner of the material is lost on a variety of plane elements. These phenomena are accounted for by assuming that:

- the fracture strain is positive semi-definite (fracture corresponds to a strain state which does not produce contraction of any material element);
- the stress state is negative semi-definite (the stress state cannot suffer tractions);
- on any principal direction where the material is compressed, the relevant coefficient of linear elongation of the fracture strain is zero;

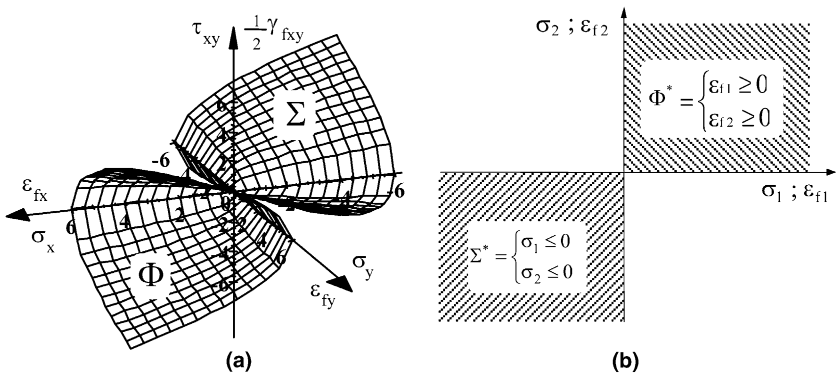


Fig. 1. The stress ($\Sigma$) and the fracture ($\Phi$) admissible domains in the spaces of tensor components (a) and of principal components (b).